Physica A 438 (2015) 81–93

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Revisited Fisher's equation in a new outlook: A fractional derivative approach

Marwan Alquran [a,b,*], Kamel Al-Khaled [b,a], Tridip Sardar [c], Joydev Chattopadhyay [a,c]

[a] Department of Mathematics and Statistics, Sultan Qaboos University, P.O. Box: 36, PC 123, Al-Khod, Muscat, Oman

[b] Department of Mathematics and Statistics, Jordan University of Science and Technology, P.O. Box: 3030, Irbid, 22110, Jordan

[c] Agricultural and Ecological Research Unit, Indian Statistical Institute, 203, B.T. Road, Kolkata 700108, India

HIGHLIGHTS

- A modified fractional Fisher is proposed to demonstrate the embedded memory index.
- We performed new analytical and numerical treatments to solve Fisher equation.
- *Phalacrocorax carbo*data set is considered as a test bed to validate our findings.
- Non-diffusive and diffusive systems are similar in a certain interval of memory index.
- The wave-like pattern observed in our findings coincides with the thought of Fisher.

ARTICLE INFO

Article history:
Received 18 December 2014
Received in revised form 10 May 2015
Available online 8 July 2015

Keywords:
Fractional differential equation
Fisher's equation
Sinc method
Approximate solutions
Generalized Taylor series
Residual power series

ABSTRACT

The well-known Fisher equation with fractional derivative is considered to provide some characteristics of memory embedded into the system. The modified model is analyzed both analytically and numerically. A comparatively new technique residual power series method is used for finding approximate solutions of the modified Fisher model. A new technique combining Sinc-collocation and finite difference method is used for numerical study. The abundance of the bird species *Phalacrocorax carbo*is considered as a test bed to validate the model outcome using estimated parameters. We conjecture non-diffusive and diffusive fractional Fisher equation represents the same dynamics in the interval (memory index, $\alpha \in (0.8384, 0.9986)$). We also observe that when the value of memory index is close to zero, the solutions bifurcate and produce a wave-like pattern. We conclude that the survivability of the species increases for long range memory index. These findings are similar to Fisher observation and act in a similar fashion that advantageous genes do.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

In dissipative dynamical systems, including physical, chemical and biological communities, non-linear reaction–diffusion (R–D) equations play a significant role to describe the dynamics of the system. The classic and simplest case of R–D equation is the so-called Fisher equation

$$\frac{\partial u}{\partial t} = \lambda \frac{\partial^2 u}{\partial x^2} + \mu u(x,t)(1 - u(x,t)) \tag{1.1}$$

* Corresponding author at: Department of Mathematics and Statistics, Jordan University of Science and Technology, P.O. Box: 3030, Irbid, 22110, Jordan.
E-mail addresses: marwan04@just.edu.jo, marwan04@squ.edu.om (M. Alquran).

http://dx.doi.org/10.1016/j.physa.2015.06.036
0378-4371/© 2015 Elsevier B.V. All rights reserved.

which is basically the conjunction of diffusion equation and the Logistic equation with birth rate μ and diffusion coefficient λ. It is important to note that Fisher [1] originally proposed the model to investigate the dynamics of the spatio-temporal propagation of a virile gene in an infinite domain. Since the pioneering work of Fisher, this equation and simple modified Fisher equation are widely used in chemical kinetics, flame propagation, auto-catalytic chemical reactions, branching Brownian motion [2], spread of invasive species [3], bacteria [4], epidemics [5] and many other disciplines.

In recent times, the researchers are interested to work on fractional nonlinear partial differential equations. The main reason for such interest in this system is due to its self-organization phenomena and memory index. Fractional derivatives provide an excellent tool for describing memory and hereditary properties of various materials and processes [6]. It is to be noted that the solution which we obtain from classical calculus is basically based on individual-level perspective. But most of the physical, chemical and biological systems possess memory based concept. To learn the behavior of such systems, we need the solution from community level. The fractional derivative has the property to provide such solutions. Although the physical meaning of fractional derivative is still an open problem, but in recent times, a considerable interest in this aspect has been put forwarded by the researchers [7,8]. It is already pointed out that Fisher's equation has been widely used in different dimension. However, from the memory based concept, the use of Fisher equation is very limited or none. We like to investigate the Fisher equation both from analytically and numerically with an aim to provide a few characteristics in terms of memory index. To capture memory index, we need a non-local operator. The fractional order differential operator is non-local and fractional order differential equations (FDEs) have all the elements to capture memory. With this backdrop, Eq. (1.1) reduces to

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \lambda \frac{\partial^2 u}{\partial x^2} + \mu u(x,t)(1 - u(x,t)) \tag{1.2}$$

subject to the initial condition

$$u(x,0) = f(x), \quad x \in (a,b), \tag{1.3}$$

and the boundary condition

$$u(a,t) = g_a(t), \qquad u(b,t) = g_b(t), \quad t > 0. \tag{1.4}$$

The fractional differential operator $\frac{\partial^\alpha u}{\partial t^\alpha}$ describes the fractional time-derivatives of order α of Fisher's equation (1.1), where $0 < \alpha \le 1$. In our study, we use the fractional derivative in the Caputo sense [9]. We shall introduce now a modified fractional differentiation operator D^α proposed by Caputo in his work on the theory of visco-elasticity [9]. The main advantage of Caputo's approach is that the initial conditions for fractional differential equations with Caputo derivatives take on the same form as for integer-order differential equations. In case of $0 < \alpha < 1$, the definition of Caputo fractional derivatives uses the information of the standard derivatives at all previous time levels. Some properties valid for integer differentiation and integer integration remain valid for fractional differentiation and fractional integration; namely the Caputo fractional derivatives and the Riemann–Liouville fractional integrals are inverse operations. For more details on the geometrical and physical interpretations for fractional derivatives of both Riemann–Liouville and Caputo types, see Refs. [9,10].

Definition 1.1 (*[11,12]*)**.** For m to be the smallest integer that exceeds α, the Caputo fractional derivatives of order $\alpha > 0$ is defined as

$$D^\alpha u(x,t) = \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)} \displaystyle\int_0^t (t-\tau)^{m-\alpha-1} \frac{\partial^m u(x,\tau)}{\partial \tau^m} d\tau, & m-1 < \alpha < m \\ \dfrac{\partial^m u(x,t)}{\partial t^m}, & \alpha = m \in N. \end{cases}$$

For mathematical properties of fractional derivatives and integrals one can consult the above mentioned references.

The present paper is an attempt to solve the fractional Fisher equation (1.2) in a new outlook from both analytical and numerical points of view. For analytical solution, we use residual power series (RPS) which enables us to find approximate solution in the form of polynomial with higher precision and less computational time. For numerical solutions, we combine two methods, namely, the finite difference method to replace the first-order time derivative of order α and Sinc-collocation method in the space direction. To the best of our knowledge, the proposed method has not been used for solving fractional Fisher equation.

The fractional order differential Fisher equation was analytically solved by many authors using Adomian and variational iterative schemes [13–16]. Recently, the authors have used Residual Power Series (RPS) method to solve fractional order partial differential equations [17–19]. The RPS method is basically based on the generalization of Taylor series expansion. The RPS method is easy to handle and more reliable than other methods used to solve fractional partial differential equations.

We also like to mention that to solve fractional partial differential equations, various numerical methods [13–16,20] were used. But the scheme the authors used are Adomian decomposition method, Variational iteration method and the local fractional expansion method. These methods have a very small convergence region. In addition to that the results thus obtained using these methods caused high-frequency oscillation, which must be filtered at each step. In general, it is difficult to specify all the terms for a series solution for problems involving fractional partial differential equations. Thus only the first few terms

Download English Version:

https://daneshyari.com/en/article/974805

Download Persian Version:

https://daneshyari.com/article/974805

Daneshyari.com