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Wetting in 1 + 1 dimensions with two-scale roughness

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HIGHLIGHTS

- Cassie–Baxter, Wenzel and intermediate wetting states.
- Hydrophobicity with two-scale roughness.
- Exact computation of surface phase diagrams in 1 + 1 dimensions.

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ABSTRACT

We show that a two-scale model in 1 + 1 dimensions enhances superhydrophobicity. The two scales may differ by a factor of order two or three, or by a large factor in a scaling limit. In both cases, we compute explicitly the macroscopic contact angles as function of the flat material contact angle and aspect ratios. In addition to the Cassie–Baxter states with air cushion below the droplet and to the Wenzel states, completely wet, there appear several mixed states with air trapped in corners.

1. Introduction

Nature has designed many different examples of superhydrophobic surfaces for plants or insects. When a water droplet is deposited on it, high values of both advancing and receding contact angles are observed. Superhydrophobicity will also provide special characteristics such as rolling motion of a deposited water drop with a very low tilting angle or rebound of the drop when impacting the surface. For such surfaces, roughness plays a key role. When deposited on such surfaces, it is expected that the drop can be in at least two different states: in contact everywhere with the solid surface, i.e. the Wenzel state [1], or in contact with the top elements of the surface, the Cassie–Baxter state, [2]. The basic idea for such systems is that Nature will minimize the free energy leading to the conclusion that if the equilibrium contact angle θ_0 corresponding to the flat surface satisfies some inequality, the drop will be in the Cassie–Baxter state leading to superhydrophobicity. In 1+1 dimensions, or in three dimensions with grooves, we can build a regular surface obtained as a periodic substrate with unit cell of parameters (a, b, c) as shown in Fig. 4. When dealing with such simple geometry, this inequality is well known and can be written as

$$\cos\theta_0 < -(1-\phi)/(r-\phi), \quad (1)$$

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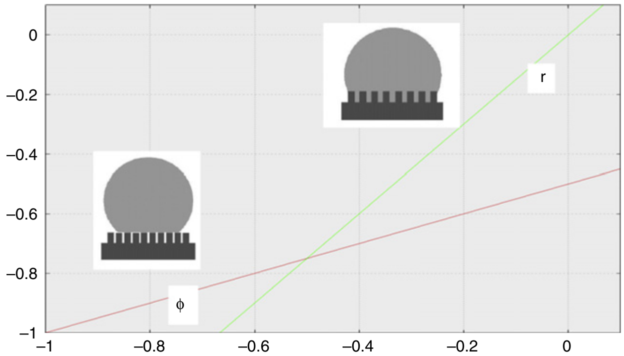

Fig. 1. Cosine of the macroscopic contact angle $\cos\theta$ on rough surface versus cosine of the flat surface contact angle $\cos\theta_0$, in Cassie–Baxter and Wenzel states regimes. The *CB* line has slope ϕ, the *W* line has slope r.

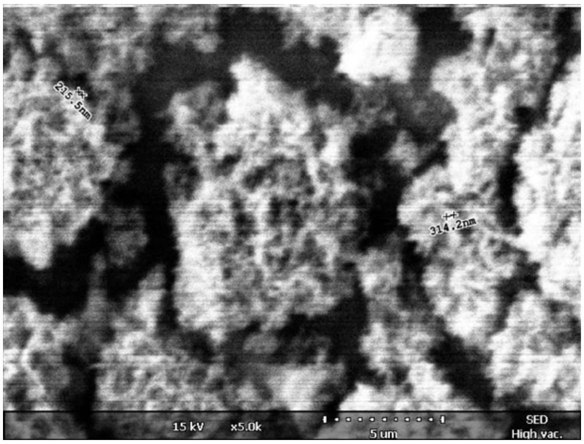

Fig. 2. A microscope view of a two-scale pattern (lotus leaf).

where r is the Wenzel roughness of the surface, defined as the total area of the surface divided by its projection, and ϕ is the covered fraction, defined as the total area at the top level divided by the total projected area. As can be easily seen, the larger r, the more the surface is likely to be in the Cassie–Baxter state. This is the reason why it is believed that a very rough hydrophobic surface can be superhydrophobic. These considerations are generally summarized in a single graph describing the cosine of the contact angle θ of a rough surface versus the cosine of the contact angle θ_0 of the corresponding flat surface, as represented in Fig. 1. Motivated by superhydrophobicity, we limit ourselves to $\theta_0 > \pi/2$, which is also required by the latter inequality (1).

For nanoparticles on top of a solid flat surface (see Ref. [3]), the same type of results are obtained with more curved lines instead of straight lines. One of the remarkable properties of these systems is that the slope at the origin ($\cos\theta_0 = 0$, $\theta = 0$) for the Wenzel regime is always the roughness r, and that the slope at ($\cos\theta_0 = -1$, $\cos\theta = -1$) is always the surface fraction ϕ. This kind of simple reasoning is based on surfaces with a single scale topography. Nature has designed remarkable hydrophobic surfaces showing 2 scales of roughness (see Fig. 2). It is the purpose of this paper to analyze in detail how a second topography scale can improve the possible superhydrophobicity of a surface. Indeed, from the previous considerations, one may expect that by adding a second scale of grooves or nanoparticles, one will increase (resp. decrease) inevitably r (resp. ϕ) leading to an improvement of the Cassie–Baxter regime, compared to a one-scale situation. Dealing often with biomimetics, several authors have considered this kind of problem and are usually going in this direction, see Ref. [4–9] just to quote a few. Enhancement of superhydrophobicity with two length scales has recently received particular attention, see for instance and not exhaustively [10–12]. However, the rules to combine the different roughnesses or surface area are not particularly clear, [13]. In a few publications moreover, [14,15], it is shown that some double scale or hierarchical structures are not even particularly favorable for a Cassie–Baxter state to emerge.

There is thus a real need for a rigorous analysis of the effect of the second scale roughness in the problem of superhydrophobicity. We address this problem here systematically in the context of the simple groove geometry (1+1 dimensions). We restrict our attention to flat air/liquid interfaces, applicable to equilibrated gently deposited macroscopic droplets as in Ref. [16] and not for instance to later stages of evaporating droplets [17]. Accordingly, the dimension of the macroscopic droplet is assumed much larger than the width and height of the grooves. Under these assumptions, the Cassie–Baxter/Wenzel approach considering states with minimum free energies does hold, as a result of the drop size being