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On the appearance of traffic jams in a long chain with a shortcut in the bulk

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HIGHLIGHTS

- We study stationary properties of TASEP on open long chains with a shunted section.
- The conditions for occurrence of traffic jams and their properties are investigated.
- Monte Carlo simulations data agree fairy well with theoretical predictions.
- The effective rates approximation and domain wall theory are applicable.
- Traffic jams are found to disappear upon shifting the shortcut downstream.

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ABSTRACT

The Totally Asymmetric Simple Exclusion Process (TASEP) is studied on open long chains with a shunted section between two simple chain segments in the maximum current phase. The reference case, when the two branches are chosen with equal probability, is considered. The conditions for the occurrence of traffic jams and their properties are investigated both within the effective rates approximation and by extensive Monte Carlo simulations for arbitrary length of the shortcut. Our main results are: (1) For any length of the shortcut and any values of the external rates in the domain of the maximum current phase, there exists a position of the shortcut where the shunted segment is in a phase of coexistence with a completely delocalized domain wall; (2) The main features of the coexistence phase and the density profiles in the whole network are well described by the domain wall theory. Apart from the small inter-chain correlations, they depend only on the current through the shortcut; (3) The model displays unexpected features: (a) the current through the longer shunted segment is larger than the current through the shortcut, and (b) the delocalized domain wall in the coexistence phase of the long shunted segment induces similar behavior even in shortcuts containing a small number of sites; (4) From the viewpoint of vehicular traffic, most comfortable conditions for the drivers are provided when the shortcut is shifted downstream from the position of coexistence, when both the shunted segment and the shortcut exhibit low-density lamellar flow. Most unfavorable is the opposite case of upstream shifted shortcut, when both the shunted segment and the shortcut are in a high-density phase describing congested traffic of slowly moving cars. The above results are relevant also to phenomena like crowding of molecular motors moving along twisted protofilaments.

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1. Introduction

The one-dimensional asymmetric simple-exclusion process (ASEP) is one of the simplest models of self-driven many-particle systems with particle conserving continuous time stochastic dynamics. The process was first introduced in Ref. [1] as a model of kinetics of protein synthesis, i.e., of the biological process of ribosome translocation along a messenger ribonucleic acid (mRNA). The totally asymmetric simple exclusion process (TASEP) is one of the rare examples of exactly solvable models with non-equilibrium steady states. On simple chains its stationary properties have been extensively studied and exactly solved in the thermodynamic limit for periodic, closed and open boundary conditions, first for random sequential update and then for a number of stochastic dynamics in discrete time: forward- and backward-ordered, sublattice parallel and parallel, see the reviews [2–5] and references therein. In the case of open chains, TASEP was shown to exhibit boundary induced phase transitions, spontaneous symmetry breaking and phase separation on the coexistence line between the low and high density phases. Recently, it has inspired a variety of modifications and extensions designed to model diverse biological problems, see, e.g., the reviews [6,7] and references therein.

One of the natural and frequently used interpretations of TASEP is given in terms of a single-lane vehicular traffic, see the reviews [3–5]. Various extensions of the basic model were devised to describe different driving conditions and drivers strategies. In this interpretation, the boundary induced first order phase transition in open chains is from free flow to congested traffic. The shock, which represents a discontinuity in the density profile, models the front of a traffic jam. The fully parallel dynamics is considered to be the most appropriate for traffic modeling and it is laid in the basis of more sophisticated update rules [8–10]. The popular Nagel–Schreckenberg traffic model [8] under maximum vehicle velocity $v_{\max} = 1$ reduces to the TASEP with parallel update.

Obviously, the search of new control mechanisms, vehicular traffic optimization, elimination or reduction of congested traffic jams are one of the important social problems, the solution of which needs mathematical modeling. The development of computational resources provided the possibility of simulating traffic flow in discretized space and time. Usually, traffic jams are observed at bottlenecks such as traffic accidents, lanes merging, or some kind of localized inhomogeneity. However, traffic jams often appear on crowded highways due to spontaneous fluctuations in the flow. An experimental verification that a jam can be generated in the absence of a bottleneck is reported in Ref. [11]. Here we confine ourselves with the stationary states of TASEP and jams which appear in the coexistence phase of a shunted segment. Characteristic of these jams is that the average position of the domain wall between the low- and high-density phases is stationary, determined by the average number of particles in the system, whereas the domain wall itself performs a symmetric random walk on the chain with reflecting boundary conditions.

The special case of a network with a section of two parallel chains of equal length inserted in the bulk of a long chain was studied in our paper [12] both analytically and by extensive Monte Carlo simulations. Since there are no exact results for TASEP on networks with junctions, we introduced effective injection and ejection rates for each chain segment and studied the possible phase structures of the system in terms of these rates. This approach, called later Effective Rates Approximation (ERA), turned out to be very effective in the study of the stationary phases of complex networks, composed of long linear chains, see, e.g. Refs. [13–19] and the recent reviews on TASEP over networks with complex geometry [20,7]. We found a coexistence phase in the double-chain segment when the head and tail chains are of equal length and in the maximum current phase. This result is interesting on its own because the set up essentially differs from the case of a simple chain coupled to reservoirs with $0 < \rho_- = 1 - \rho_+ < 1/2$, when coexistence with a stationary position of the domain wall (shock) has been proved to exist. Recently, we have studied also the dependence of the phase in the double-chain segment on its position in a long but finite network [21].

Two models of TASEP on open chains with a zero-length shortcut in the bulk were introduced in Ref. [22]: model A for molecular motor motion, and model B for vehicular traffic. In the former case the molecular motors walk along a filament which is twisted so that a motor may jump with probability q between two sites, which are far away along the filament but close in real space. Model A was reexamined in our recent paper [23]. It turned out that the authors of Ref. [22] used an unfounded approximation due to which the coexistence phase in the shunted chain, in the case of maximum current through the network, was not detected. Our theoretical analysis, based on the ERA, has shown that the second (shunted) segment can exist in both low-density and high-density phases, as well as in the coexistence (shock) phase. Numerical simulations have demonstrated that the last option takes place in finite-size networks with head and tail chains of equal length, provided the injection and ejection rates at their external ends are equal and greater than one half. Then the local density distribution and the nearest-neighbor correlations in the middle chain correspond to a shock phase with completely delocalized domain wall, as is the case studied in Ref. [12]. Upon moving the shortcut to the head or tail of the network, the density profile takes shape typical of a high- or low-density phase, respectively, in complete parallel with the results in Ref. [21] obtained for networks with a double-chain section.

There are some other conditions for the occurrence of a shock phase in the shunted segment, as well as in the shortcut, when it is long enough. For example, when the head and tail segments are in low- or high-density phases, they carry a current less than the maximal one, $J < 1/4$, which implies that their bulk densities are given by the two solutions of the equation $\rho_{\pm}(1-\rho_{\pm}) = J$, where $\rho_- = 1-\rho_+ < 1/2$. Therefore, these segments can be: (i) both in a low-density phase, with bulk density $\rho_{bulk}^{(1,3)} = \rho_-$, (ii) both in a high-density phase, with bulk density $\rho_{bulk}^{(1,3)} = \rho_+$, and, (iii) in the symmetry-broken case, the head segment is in the low-density phase with $\rho_{bulk}^{(1)} = \rho_-$, while the tail segment is in the high-density one with