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Quantum spatial-periodic harmonic model for daily price-limited stock markets

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HIGHLIGHTS

- The price of a price-limited stock is presumed oscillating and damping in a spatial-periodic harmonic oscillator potential well.
- There is negative non-linear relation between the volatility and trading volume of a stock.
- Price limit will abnormally increase volatility if within a certain regime of trading volume.

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ABSTRACT

We investigate the behaviors of stocks in daily price-limited stock markets by purposing a quantum spatial-periodic harmonic model. The stock price is considered to be oscillating and damping in a quantum spatial-periodic harmonic oscillator potential well. A complicated non-linear relation including inter-band positive correlation and intra-band negative correlation between the volatility and trading volume of a stock is numerically derived with the energy band structure of the model concerned. The effectiveness of price limit is re-examined, with some observed characteristics of price-limited stock markets in China studied by applying our quantum model.

1. Introduction

Stock market, as one of the most important financial instruments, plays an unshakable role in the basic research of economics and finance. After the noteworthy work of linking economic research with the fundamental concepts and methods of statistical physics in the 1990s [1,2], econophysics has soon burgeoned as a new interdisciplinary area, from which quantum finance [3] is then specifically introduced for applying quantum physics to finance [4–11]. With the help of quantum mechanics, rather than the wide-known classical oscillator model for stocks introduced in Ref. [12], it is recently discovered that a single stock should be treated as a quantum harmonic oscillator [13] not only excited by external information while damping to its ground state but also holding a persistent small scale of fluctuation; a stock index should be treated as a quantum Brownian particle with ensemble of stocks as a thermal reservoir [11], and a quantum Brownian model is introduced in order to explain the fat tail phenomena [14] and long-term non-Markovian features [15] of stock indices by applying the theory of quantum open systems [16]. Combining physics models with financial tools, we are able to study the underlying physical concepts of finance and economics and handle with financial problems more effectively.

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It is worth noting that the stock markets of China – an economically burgeoning developing country – are much more concerned and studied in recent years [17–19]. One of the particularities of the stock markets in China is daily price limit, i.e., the daily increase/decrease ratio of one stock price is confined to ±10% (±5% for special treated stocks) [20–24]. After reaching the limit, transaction of the stock is not paused but limited unidirectionally, and is thus strongly biased to sellers/buyers. We noticed a recent "incident" relevant to price limit in the stock markets of China, that on Aug. 16, 2013, a so-called fat finger transaction incident caused by China Everbright Securities led to an unusual fluctuation of the stock markets. The stock index of Shanghai Stock Exchange rose by +5.62% with more than thirty stocks reaching their +10% limit promptly, and then fell down in just 1 min. The fluctuation was so large that it was considered to be statistically abnormal. As the "incident" involves the daily price limit however, this eccentric phenomenon requires more sophisticating investigation. Since the effectiveness of price limit on stock markets is still doubted and argued about, we regard with attention that the daily price limit remains to be a valuable issue, on which our research will further help us to reconsider the applicability of price limit and predict its further influence on stock markets.

In this paper, we investigate the price-limited space of stock price with the help of quantum energy band theory by introducing a spatial-periodic harmonic oscillator potential well. A theoretical model is obtained, revealing the existence of an energy band structure in the price-limited space and that the energy band structure will introduce a complex relation between the volatility and trading volume of a stock. From detailed numerical solution of the spatial-periodic harmonic model, an exact non-linear relation between the volatility and trading volume is derived, which demonstrates that not only ordinary inter-band positive correlation but also abnormal intra-band negative correlation is contained in the complicated relation. The ability for the daily price limit to control volatility is then inspected; it is found that price limit will abnormally increase the volatility if within a certain regime of the trading volume. It is suggested that the existed schedule for price limit needs to be reconsidered for improvement. Using our model, some observed peculiar features (including the fat finger "incident") of the price-limited stock markets in China can also be explained qualitatively. We expect this interdisciplinary physical model to contribute to the development of econophysics as well as quantum finance.

2. Quantum spatial-periodic harmonic oscillator potential well

In 1933, a damped harmonic oscillator model [12] was presented. As one of the most important models in economics, it presumes that on account of stock trading and its auxiliary expense, the price of a single stock should oscillate and dissipate to equilibrium like a damped harmonic oscillator while being impelled by time-varying information outside [12]. This model provides an intuitive point of view by introducing physical methods into financial problems. However, according to Ref. [25], it is found that the classical model cannot explain why there exists a persistent small scale of fluctuation of the stock price. Instead, a quantum harmonic oscillator model is introduced [13]. A quantum harmonic oscillator model ensures that the volatility of a single stock is always non-zero even if there is no information to impel the stock price and the oscillator decays into the ground state: the probability distribution $|\varphi(x)|^2$ of position x (logarithmic stock price) of the ground state in a harmonic potential well $V(x) = m\omega^2x^2/2$ is a Gaussian distribution (see Fig. 1(a)), i.e., $|\varphi_0(x)|^2 = \pi^{-1/2}\beta \exp\left(-\beta^2x^2\right)$ with $\beta = \sqrt{m\omega/\hbar}$ (in general, one has $|\varphi(x)|^2 = \pi^{-1/2}\beta \exp\left(-\beta^2x^2\right) H_n^2(\beta x)/2^n n!$ with H_n the Hermite polynomials) [26]. The physical quantities m, ω, and $\hbar$ separately represent the capital, the characteristic frequency, and the uncertainty of irrational transaction of the stock [11,13]: apparently, the inertia of capital against price fluctuation is analogous to the mass m; the characteristic frequency, i.e., the inverse of the response time against excitation from outside information, is analogous to the oscillating frequency ω; and more crucially, the uncertainty (or the so-called *generalized uncertainty* for discrete price [27]) of irrational transaction is analogous to the Planck constant $\hbar$, which acts as a link between the intrinsic probability of quantum mechanics and persistent fluctuation caused by irrationality in stock markets [11].

It is also known that among the energy levels of a quantum harmonic oscillator $E = (n + 1/2)\hbar\omega$ one will certainly find a non-zero ground energy $\hbar\omega/2$. Considering that trading volume is almost positively and linearly related to volatility [28], and that the volatility $\sigma_x^2 = \int |\varphi(x)|^2 x^2 dx$ of a quantum harmonic oscillator is equal to $E/m\omega^2$ [26], we can treat the energy E as trading volume then. In the ground state, the stock remains a standstill. Its price should be determined and equal to its real value if all transactions are rational ($\hbar \to 0$), then no transaction takes place any more ($E \to 0$). Thus, the irrational transaction of a stock also gives rise to a persistent non-zero trading volume, which satisfies our presumption.

Since the daily fluctuation of stock price is confined to a region of ±10% in the stock markets of China, a reconsideration for the quantum harmonic oscillator model is required. Before, the limit was considered as a cutoff boundary condition of x, i.e., the oscillating particle is confined to an infinite square well potential $-d/2 \le x \le d/2$ as the probability of finding the price out of the width d is absolutely zero. However, this cutoff boundary condition also implies $|\varphi(-d/2)|^2 = |\varphi(d/2)|^2 = 0$ to fit the requirement of continuity, thus the probability of reaching the limit is zero, which looks invalid for practical stock markets. To modify the boundary condition, $|\varphi(-d/2)|^2 = |\varphi(d/2)|^2$ is required for symmetry, but is not necessary to be zero. The boundary condition thus indicates $\varphi(-d/2) = e^{-ikd}\varphi(d/2)$. By making continuation of $\varphi(x)$ out of the limited width d to infinity and assuming it has a periodic pattern, one has

$$\varphi(x) = e^{-ikd}\varphi(x + d), \tag{1}$$

which satisfies the one-dimensional Bloch theorem with k a Bloch wave number (phase information of a stock) [29]. This periodic boundary condition introduces a spatial-periodic harmonic potential well $U(x)$ (see Fig. 1(b)). In $-d/2 \le x \le d/2$,