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# A non-extensive statistical model for time-dependent multiple breakage particle-size distribution

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## HIGHLIGHTS

- Non-extensive statistical model.
- Fractionary time evolution.
- Particle size distribution.
- Multiple breakage processes.

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## ABSTRACT

A general formulation for the statistical description of time-dependent multiple particle breakage processes is presented in terms of a purposely constructed dimensionless quantity that contains the main physical magnitudes involved in the problem. The approach combines the Tsallis non-extensive entropy with a kinetic equation with fractionary index for the time evolution of the size/mass of the fragments. The obtained distribution function is tested by fitting some experimental reports. It is found that the better adjustment corresponds, in all cases, to values of the time index equal or below 0.6, whereas the parameter of nonextensivity ranks between 1 and 2, as previously reported in other studies involving some kind of fragmentation. The work could be the first example of a non-extensive maximum-entropy statistical description based on a purposely constructed dimensionless quantity, as well as of the derivation of a fragment size distribution function explicitly dependent on measurable system variables. As a result, the role of quantities such as viscosity, velocity gradient and others becomes explicit in the formulation.



## 1. Introduction

The multiple breakage of crystals occurs during distinct research and technological processes such as combustion [1–3], as well as milling and destruction (for very recently reports see, for instance, Ref. [4]). It is, actually, a very complex phenomenon in which the quantity known as fragment size distribution function (FSDF) is relevant.

The problem of finding the time-dependent FSDF (TDFSDF), corresponding to an event of multiple solid breakage, as a function of the main macroscopic measurable variables is of key interest in many areas. Some experimental reports on breakage rates [5], and on size distribution during particle abrasion [6,7] have been published. A number of approaches and models

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have been elaborated to describe fragmentation phenomena and size distribution of fragments. The usual theoretical approach appeared, in a much complete form, in the work by Hill and Ng [8], and more recently in several contributions [9–19]. In the majority of these cases the ultimate goal is to achieve a comprehensive coincidence with experiments by providing a suitable fitting of the practical data. However, up to our knowledge the distinct approaches presented so far have not constructed their formalisms starting from first principles such as the fundamental maximum-entropy (MaxEnt) one.

The aim of the present work is to propose a statistical model of the multiple breakage of solid particles which can provide a general physical ground of its static and time-dependent behaviors. As it has been already discussed, the process of violent fractioning leads to the presence of long-range correlation between all the parts of the fragmented object [20]. In consequence, the usual Boltzmann–Gibbs statistics is not a correct choice. Therefore, our formalism relies on the use of the Tsallis non-extensive statistics [21–24], which has been proved to yield good results when applied to modeling problems like fragmentation [20,25], cluster formation [26] and particle size distribution in ceramic matrix agglomerations [27].

Multiple breakage exhibits a particular temporal evolution which could be characterized by saying that it has a non-typical time scaling. In order to demonstrate such assertion and properly provide a time-dependent description, our treatment proposes the suitability of a *fractal-like* kinetics along the lines of the proposal appearing in Ref. [28]. The article is organized as follows. Section 2 contains details of the model. The results of its application to fitting several available particle-size distribution data are presented and discussed in Section 3 and, finally, the conclusions of the work appear in Section 4.

## 2. Model

If progressive particle fragmentation takes place within a liquid environment, then quantities such as shear rate and viscosity can be some of the macroscopic variables mentioned above. On the other hand, since the multiple particle breakage involves the effect of inertial impact with container walls or with another particles, it is possible to assume that the fragment mass and the shear rate must appear in the distribution function. The effect of attrition also determines the fragment size, so we also need to take into account the influence of viscosity. Besides, both inertial impact and viscosity depend on fragment concentration. All these variables appear in Ref. [5] as the main factors governing the FSDF. Hence, the considered physical quantities are: $m$—the mass of the fragments; $\dot{\gamma} = \nabla v$—the shear rate (gradient of the velocity); $\eta$—the liquid viscosity, and $n$—number of fragments per unit volume. Since the basic dimensions entering this problem are mass ($M$), length ($L$), and time ($T$) (all positive, as noticed); then, according to the Vaschy–Buckingham theorem [29], the law relating all those variables can be transformed into a single equation that includes, in this case, only one dimensionless variable. In our case, this variable is

$$\xi = \frac{m\dot{\gamma}n^{1/3}}{\eta}; \tag{1}$$

Thus, the problem of finding the distribution of volume (mass), or FSDF, can be formulated as the derivation of the distribution function of the dimensionless variable $\xi$. This can be accomplished using basic Physics principles such as the Second Law of Thermodynamics, which is nothing but a maximization of the system's entropy. However, as mentioned above, the use of the Boltzmann–Gibbs (BG) entropy is not suitable in this case [20]. When long-range correlation is relevant, the use of the Tsallis entropy [21] reveals to be convenient. In its continuous version, the form of this entropy (in units of the Boltzmann constant) is:

$$S_q = \frac{1 - \int p^q(\xi)\,\mathrm{d}\xi}{q-1}. \tag{2}$$

In this expression, $p$ is the probability density function. The quantity $q$ is known as the "degree of non-extensivity" and, in principle, can take any real value. It is possible to verify that, under the normalization condition $\int_0^A p(\xi)\,\mathrm{d}\xi = 1$, the limit when $q \to 1$ leads to the BG entropy.

The search for a maximum of $S_q$ must include some constraints. One of them is, precisely, the normalization of the distribution, which is usual in the analysis of the BG case. The second constraint is not that usual. In this case, it is customary to impose the finiteness of the so-called $q$-mean value, also named as the first-order $q$-moment:

$$\mu = \int_0^A \xi\, p^q(\xi)\,\mathrm{d}\xi. \tag{3}$$

As in the case of normalization, integration limits should include a maximal fragment size (mass), included in the upper limit for the variable ($\xi = A$), and a minimal one that corresponds to the situation when the breakage process cannot yield fragments of smaller dimensions. In order to simplify the treatment, our treatment sets the lower integration limit as zero. Under the constraints mentioned, the problem of finding the maximum of the entropy is no other than a Lagrange multipliers one. Without loss of generality, we shall proceed as in Ref. [20] and write $\mu = 1$, thus setting this quantity as a kind of reference value of the $q$-moment. Under these circumstances, the – static – distribution function for the dimensionless variable $\xi$ will be

$$p(\xi) = (2-q)^{\frac{1}{2-q}} \left[1 - \frac{q-1}{2-q}(2-q)^{\frac{1}{2-q}}\,\xi\right]^{\frac{1}{1-q}}. \tag{4}$$