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Monte Carlo tests of small-world architecture for coarse-grained networks of the United States railroad and highway transportation systems

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HIGHLIGHTS

- We use Monte Carlo randomizations to test US transportation infrastructure data for small-worldness.
- We compare the US railroad network, US highway network, and a simple planar network.
- Transportation networks have small-world architecture relative to global randomizations.
- Transportation networks do not have small-world architecture relative to planar randomizations.
- The small-world structure of transportation networks owes more to their planarity than to the topology of transportation links.

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ABSTRACT

Several studies have shown that human transportation networks exhibit small-world structure, meaning they have high local clustering and are easily traversed. However, some have concluded this without statistical evaluations, and others have compared observed structure to globally random rather than planar models. Here, we use Monte Carlo randomizations to test US transportation infrastructure data for small-worldness. Coarse-grained network models were generated from GIS data wherein nodes represent the 3105 contiguous US counties and weighted edges represent the number of highway or railroad links between counties; thus, we focus on linkage topologies and not geodesic distances. We compared railroad and highway transportation networks with a simple planar network based on county edge-sharing, and with networks that were globally randomized and those that were randomized while preserving their planarity. We conclude that terrestrial transportation networks have small-world architecture, as it is classically defined relative to global randomizations. However, this topological structure is sufficiently explained by the planarity of the graphs, and in fact the topological patterns established by the transportation links actually serve to reduce the amount of small-world structure.

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1. Introduction

Human transportation systems are vital components of a nation's economy, promoting the movement and delivery of persons, goods, and services [1]. There are also collateral movements of undesirable factors such as chemical toxins [2], human diseases [3], and invasive species [4–8], each carrying a cost. For example, it is estimated that invading alien species bring economic losses of nearly $120 billion per year in the United States [9]. Partly because the stakes are high in this new era of globalism, there is considerable interest in understanding how easily things move long distances through a transportation system, and the extent to which the infrastructure promotes local contagion and mixing.

Networks are useful for the study of transportation systems. The mathematical field of graph theory, which treats networks as mathematical objects, is considered to have arisen in 1735 with Euler's classic transportation problem involving the Konigsberg bridges [10]. Since then, an extensive literature has accumulated regarding the organization of human terrestrial transportation networks, reviewed in Refs. [1,11–13]. Many transportation networks seem to share some basic topological features. Some of this structure is thought to emerge as a balance between minimizing the costs of constructing and maintaining the transportation systems versus the ease of traversing them [14–16]. For example, many studies have concluded that terrestrial transportation networks display a small-world architecture: urban street networks [17] urban public transportation networks [15,18,19], and railroads [20–23].

The small-world phenomenon was first characterized for a social system [24] where it was shown that US acquaintance networks linked people together in roughly six steps. This was a remarkable finding given the large size of the US population and the fine-scale complexity of social relationships. The random graph theory of Erdős and Rényi [25–27] had already shown that large random networks were easily traversed by virtue of shortcuts through the network, especially compared to regular networks such as grids or lattices. Several decades later, physicists Watts and Strogatz [28] recognized the ubiquity of small-world networks and formalized their quantitative analysis. They showed that many naturally occurring complex networks share features of both regular graphs (high local organization) and random graphs (easily traversed), this by virtue of a small number of random re-wirings that introduce shortcuts across an otherwise highly organized network. They introduced a measure of fine-scale structure, where the clustering coefficient of node x (C_x) is the probability that nodes y and z also are linked given that they both link to node x. The average clustering coefficient of a small-world network should be much greater than that of a random network. They measured the ease of traversability by the characteristic path length (L) which is the average of the all the shortest path distances between node pairs in a network. If small-world networks are as easily traversed as random graphs, Erdős and Rényi theory [25–27] indicated that path lengths should scale with the log of the graph size ($L = \log(N)/\log(z)$), meaning that even large increases in the number of nodes leads to only modest increases in the average number of links separating nodes.

The success of the small-world network model is indisputable, having been identified in complex systems ranging from actor networks, brain neural networks, to cellular networks. It has been less clear, however, exactly how best to judge if a network displays significant small-world structure. The Watts and Strogatz [28] small-world model was defined by its boundary conditions, regular and random graphs. But it is worth asking, are these the correct boundary conditions for the analysis of all complex networks? Some have suggested that Erdős–Rényi random graphs, i.e., those that are globally randomized (here, ER graphs), are not always the best models for testing structure in real networks [29,30]. For one thing, ER random graphs lack clustering; the probability that two randomly-chosen nodes are linked and complete a triangular relationship is simply the probability p that they are linked, so the expectation of the clustering coefficient $C = p$. So just how much clustering must one see in order to judge that a real network has small-world structure? The threshold is rather low relative to ER random graphs. Moreover, it is worth asking what is being tested when we compare an ER random graph to a terrestrial transportation network, for example, which is for the most part a planar graph. The ER random graph would include edges between spatially disjunct nodes that could not possibly form links in the real world (e.g., a direct link between cities on opposite coasts of the United States without any intervening intersections). Then does a significant finding indicate something about transportation networks in particular, or does it simply highlight the differences between spatially constrained planar graphs versus unconstrained ER random graphs?

The literature is quite varied when it comes to statistical tests of small-worldness in terrestrial transportation networks, but all indicate small-world structure, reviewed in Ref. [31]. Some studies have concluded small-world structure based mainly on the observation of high clustering and small diameter or characteristic path length [19,20]. Many compare the observed structure to the theoretical expectations for an ER globally random graph as outlined by Watts–Strogatz [21,22]. Few studies compare transportation networks to random planar graphs. Masucci et al. [32] examined the London street network and made a random planar graph with as many nodes but distributed as a Poisson process and randomly linked; this single random model was then compared to the real network. Cardillo et al. [29] compared observed planar graphs with randomly-generated planar graphs of two types: minimum spanning trees and greedy triangulations. And some have developed or borrowed alternative but related metrics to quantify small-worldness including network efficiency [15] and spatial autocorrelation methods [31].

A less-explored approach would be to compare the observed transportation network to multiple spatially constrained, planar randomizations of the same network. One study of the US highway system [33] generated such randomizations, but this was for assessing scale-free structure, not small-world structure. Planar randomizations can be performed through a Monte Carlo simulation [34] by allowing random edge formation only between spatially adjacent nodal entities, such as between counties or states that share a border. In research on brain networks [35], Monte Carlo randomizations were