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Fragment distributions for brittle rods with patterned breaking probabilities

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ABSTRACT

We present a modeling framework for 1D fragmentation in brittle rods, in which the distribution of fragments is written explicitly in terms of the probability of breaks along the length of the rod. This work is motivated by the experimental observation of several preferred lengths in the fragment distribution of shattered brittle rods after dynamic buckling [J.R. Gladden, N.Z. Handzy, A. Belmonte, E. Villermaux, Dynamic buckling and fragmentation in brittle rods, Phys. Rev. Lett. 94 (2005) 35503]. Our approach allows for non-constant spatial breaking probabilities, which can lead to preferred fragment sizes, derived equivalently from either combinatorics or a nonhomogeneous Poisson process. The resulting relation qualitatively matches the experimentally observed fragment distribution, as well as some other common distributions, such as a power law with a cutoff.

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1. Introduction

The fragmentation of a brittle solid is an everyday example of probability and apparent randomness in a physical system: a glass or plate dropped on the floor usually shatters "into a million pieces" – very few large ones and many more small ones – and experience shows that no two plates break in the same way. While the phenomenon of a single crack or fracture has now advanced in many aspects to the stage of a general agreement between a well-developed mathematical theory and many careful experimental studies (see e.g. Refs. [2,3]), the mathematical study of fragmentation, which can be thought of as the multiple fracture limit, is still at a developmental stage, with no universally accepted theoretical approach to the wealth of empirical information [4–7]. The process of fragmentation has probably been known since prehistoric times, and observations of the phenomenon can be traced back to the ancients, as illustrated by this passage from the Iliad [8]:

But fierce Atrides wav'd his sword and strook
Full on his casque: the crested helmet shook;
The brittle steel, unfaithful to his hand,
Broke short: the fragments glitter'd on the sand.

The fundamental question of fragmentation is this: how does a single solid object break into many pieces? Certainly a number of different but interrelated physical mechanisms are involved in this breaking, and there may be different kinds of fragmentation [4,7], for instance ductile vs brittle [9,10], kinetic energy-dominated vs static stress-dominated [11], or even the fragmentation of a thin brittle coating attached to an easily deformed unbroken substrate [12]. Mathematical approaches to modeling this process, in order to obtain observed quantities, such as the mean fragment size or the fragment distribution, fall into two broadly defined categories: those starting from a mechanical perspective of the stressed material, and those aiming to derive a specific functional form for the distribution (typically power law) in a post hoc manner. These

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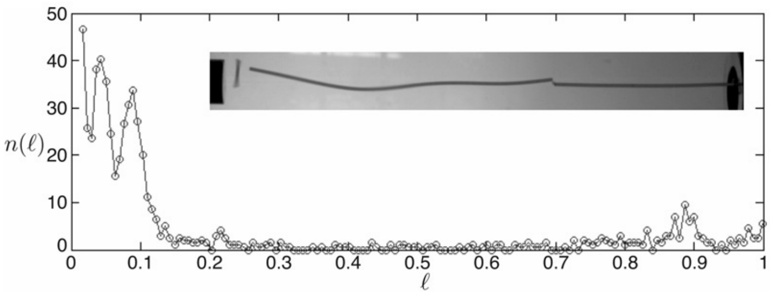

Fig. 1. Measured number density of fragments $n(\ell)$ from one fragmentation data set for the dynamic buckling of a brittle rod (here spaghetti) [1]. The inset shows an experimental image of an event representing the observation that the rod tends to break either near the top or near the bottom [29].

latter approaches are usually motivated by the many experimental studies which have reported power laws in measured fragment distributions, such as those observed in the impact fragmentation of brittle glass rods [5,13], disks, or spheres [14]. A power law dependence, with an exponential cutoff at larger sizes, is also seen in explosively fragmented distributions [15]. Some of the other physical situations, in which power law particle distributions have been reported, include ice floe size distribution in the Arctic [16], meteor shower mass distributions [17], and the size distribution of mercury drops which break into many pieces upon impact [18]. Post hoc models used to produce power laws in fragmentation include many suggesting the iterative breaking of a body [19,18,20]. The models range from simple to complex, but do not necessarily include a physical motivation. In contrast, Astrom described iterative breaking in a model motivated by the branching and merging of cracks along a fracture surface [6,21]; some similar results were given earlier by Gilvarry, but based on different considerations [22].

The interest in power laws is that they are indicative of a self-similar process, which can suggest a universal theory; however, there is no single exponent which is observed. Moreover, there are also several reports of two scaling regimes (two different power laws); this has been related in some models to a transition from 1D to 3D effects [13], and indeed a recent model treats dynamic fragmentation for D (arbitrary) dimensions [6]. The value of the exponent is usually a free parameter in these models. While the fragmentation process shares several similarities with turbulence in fluids [23], the fact that there is no universal power law exponent is a major difference, and so far this analogy has not been useful.

One early approach to modeling fragmentation was the work of Kolmogorov [24], inspired by the measurement of a log-normal distribution of fragment sizes produced by grinding. Kolmogorov used a few mathematical parameters to describe the continual grinding of larger particles into smaller particles. The primary requirement in his approach is that the fragmentation process reaches a condition where it is independent of particle size, independent of the fragmentation of other particles, and independent of the starting time—this last point implies that the probabilities are independent of the history of the particle in question. Under these assumptions, and two others involving the size and integrability of the expected number of particles resulting from a single particle per unit time, Kolmogorov deduced that the long time limit of the fragment distribution was log-normal.

Another historical strand goes back to the 1947 paper by Mott, motivated by military questions on the fragmentation of shell cases [25]. This approach originates from more physical constraints, treating local deformations and stress release after a break occurs. The literature in this area includes energy-based models in the dynamic regime (due to impact or stress-wave loading) [7,26], as compared to more of a flaw-dominated approach [27]. Many developments have been made in the geophysical community, particularly regarding the fragmentation of rocks due to geological or blasting processes (for an overview see Ref. [7]).

More recently, another approach to fragmentation was taken by Audoly & Neukirch [28], in which the dynamics of curvature after an initial break in a 1D brittle solid (in this case spaghetti) is described by a dispersive equation. In their model, the breaking is governed by the intersection of reflected self-similar solutions to this equation, which drives a focusing of the curvature, leading to breaking events. The dynamic spreading of fragmentation probability was confirmed by their experiments, in which a first break leads to a second.

In contrast to the focus on scale-invariant distributions, a recent experiment on the dynamic buckling and fragmentation of thin brittle rods found fragment distributions with two peaks, indicating preferred fragment sizes [1]. These lengths apparently originated with initial sinusoidal buckling of the rod, leading to local maxima in the fragment distribution near 1/2 and 1/4 of the buckling wavelength, see Fig. 1. An explanation was proposed, based on the assumption that breaks in the rod were more likely to occur around the points of maximum curvature, although there were many observations of breaking when the spaghetti did not break at every maximum (Fig. 1, inset). The speculative conclusion was that the distribution of fragments was being determined primarily by the initial stress distribution, rather than by a sequential, multiplicative process [1]. This indication that coherent patterns in the deformation can play a role in determining the fragment distribution provided the impetus for the present work.

The challenge posed by these observations to mathematical modeling, was the existence of multiple peaks in the fragment distribution—as opposed to a self-similar, scaling law or a single preferred fragment size. While it is clear that the coherent pattern comes in some way from the spatial distribution of stress, deformation, and perhaps other fields in

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