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Exact traveling wave solutions of the van der Waals normal form for fluidized granular matter

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HIGHLIGHTS

- We model the VdW form for fluidized granular media to study the phase separation.
- The Painlevé analysis is discussed to illustrate the integrability of the model.
- We use analytical methods to solve the PDEs and discuss stability of waves.
- The dispersion properties of the model equations are studied.
- The results show that the solutions introduce solitary waves of different types.

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ABSTRACT

Analytical solutions of the van der Waals normal form for fluidized granular media have been done to study the phase separation phenomenon by using two different exact methods. The Painlevé analysis is discussed to illustrate the integrability of the model equation. An auto-Bäcklund transformation is presented via the truncated expansion and symbolic computation. The results show that the exact solutions of the model introduce solitary waves of different types. The solutions of the hydrodynamic model and the van der Waals equation exhibit a behavior similar to the one observed in molecular dynamic simulations such that two pairs of shock and rarefaction waves appear and move away, giving rise to the bubbles. The dispersion properties and the relation between group and phase velocities of the model equation are studied using the plane wave assumption. The diagrams are drawn to illustrate the physical properties of the exact solutions, and indicate their stability and bifurcation.

1. Introduction

Granular materials include sand, sugar, crushed coal, cereals, pills, cosmetics, and asteroids. The transport, mixing, segregation of granular materials is important in the mining, agricultural, metal, food, and energy industry, the majority of the products are in granular rather than liquid form. The understanding of static and dynamic properties in this form of matter is crucial in many aspects of industrial processes ranging from pharmaceuticals to civil engineering, as well as in some basic physical phenomena such as those studied in geophysics.

The study of fluids composed of inelastically colliding particles has attracted a lot of attention in the last few years. This interest has been prompted and simulated by the attempts to understand the motion of granular media in the so-called fluidized granular regime, in which the granular matter fluidized by continuous energy injection, exhibits a variety

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of phenomena that resemble those of molecular fluids. In this regime, one can think of the grains of the granular material as similar to the molecules of a fluid and try to extend the methods of kinetic theory in order to describe the behavior of the medium. This analogy has been used by several authors to derive continuum hydrodynamic-like equations [1]. The main difference with molecular fluids is that, at collisions, grains dissipate kinetic energy into the internal degrees of freedom of the grains. Hence, energy must be supplied continuously to sustain a fluidized regime. Experimentally, energy is usually injected through vibrating walls or by the gravitational field.

The inelastic hard sphere (IHS) model does simplify very much the interactions of grains. In this model, grains are described by smooth spheres of equal diameter, σ, that have only translational degrees of freedom. The collisions are instantaneous events and the energy dissipation in collisions is simplified to a description with only one parameter, the restitution coefficient, α. In a collision of two particles characterized by collision rules, numbered 1 and 2, their pre-collisional velocities $\mathbf{v}_1 = \frac{\mathbf{p}_1}{m}$ and $\mathbf{v}_2 = \frac{\mathbf{p}_2}{m}$ are changed instantaneously to their post-collisional values $\mathbf{v}'_1$ and $\mathbf{v}'_2$ according to

$$\mathbf{v}'_1 = \mathbf{v}_1 - \frac{1+\alpha}{2}(\hat{\boldsymbol{\sigma}} \cdot \mathbf{V})\hat{\boldsymbol{\sigma}} \tag{1}$$

$$\mathbf{v}'_2 = \mathbf{v}_2 + \frac{1+\alpha}{2}(\hat{\boldsymbol{\sigma}} \cdot \mathbf{V})\hat{\boldsymbol{\sigma}} \tag{2}$$

where $\mathbf{V} = \mathbf{v_1} - \mathbf{v_2}$, m being the mass of the grains and $\hat{\boldsymbol{\sigma}}$ is the unit vector points along the line joining the two colliding particles. It is convenient to define the dissipation coefficient $e = (1-\alpha)/2$ which vanishes when collisions are elastic.

In Ref. [2] authors studied the system using molecular dynamic simulations of the IHS, they found that for larger dissipation ($e = 0.02$) a spatial instability is observed: the system exhibits the coexistence of two fluid phases, characterized by different densities. Initially the fluid remains horizontally homogeneous (in the x-direction), and suddenly a bubble (defined as a low density region) appears and grows until it achieves its final size. Afterwards, the system remains stationary with the two phases coexisting. The system behaves differently, according to the value of the dissipation parameter e, such that for small dissipation parameter, four bubbles are created in the fluid with no apparent metastable time. Two of them merge into a single one, and later on the smallest one disappears or evaporates. After a transient time, the two remaining bubbles evolve slowly, one of them growing while the other one decreases. Densification fronts, created with the bubbles, are also seen.

We consider a two-dimensional system of grains on a horizontal surface with friction ignored, placed in a box with a large aspect ratio (see Ref. [2]). Henceforth, we will refer to the horizontal x and vertical y directions as the long and short directions, respectively; the system is periodic in the horizontal direction. The top wall reflects grains elastically while the bottom one injects energy into the system by means of vertical sinusoidal vibrations at frequency ω and amplitude A. The collisions with the wall are elastic with no tangential friction, thus conserving horizontal momentum. We define the granular temperature, like in molecular fluids, to be proportional to the kinetic energy per particle in the reference frame of the system.

We emphasize that both collisions with the walls and between the grains conserve horizontal momentum. Grains have only translational degrees of freedom, and there is no tangential friction between the grains at collisions. The inelastic hard sphere model has been widely studied and reproduces well many of the observed phenomena in granular fluids at moderate densities, where rotation is not fundamental (see, for example, Refs. [1–6]). The dimensions are chosen such that the diameter σ and mass m of each disk are unity. Also, taking the wall temperature as one, energy units are fixed. Under these conditions, the system is completely defined by the total number of grains N, the aspect ratio $\lambda = \frac{l_x}{l_y} \gg 1$, the global number density $n_0 = N/l_x l_y$ where N is the total number of grains.

The general solitary wave solutions for an inviscid van der Waals model were derived and investigated in Ref. [6], in the cases of both inviscid flows and flows without friction. In the mentioned paper, the system can be well described by two Korteweg–de Vries equations in the quasi-sonic limit. In the present study, the viscosity is taken into account, and the appearance of various kinds of solitary waves undoubtedly plays an important role in the description of the behavior of the granular materials flows. However, in our opinion, the study of inviscid fluid flows is no less important as it allows a deeper understanding of the appearance of complex structures in the fluidized granular matter.

The study of exact solutions of the nonlinear partial differential equations has become one of most important topics in mathematical physics. In the past decades, various powerful methods like the Inverse scattering method, variable separation approach and Homogeneous balance method were used. But, in recent years, much research works has been concentrated on the Cole–Hopf transformation method, the Jacobi elliptic method, Adomian method, and the various extensions of the Tanh-expansion method [7–12].

The paper is organized as follows. In Section 2, we describe the problem macroscopically, and introduce the basic equations governing the fluidized granular flow. In Section 3, we first present the model equation, and then we will make a suitable transformation to get its solutions. In Section 4, we present explicit Painlevé test for the model equation. In Section 5, we solve the model equation analytically by using two different methods. In Section 6, we discuss some fundamental properties of the model such as the dispersion. In Section 7, we finally introduce the necessary discussion and conclusions.