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# Pricing foreign equity option with stochastic volatility

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## HIGHLIGHTS

- We propose a general foreign equity option pricing framework.
- The time-changed Levy processes are used to model the underlying assets price in our model.
- The closed form pricing formula is obtained through the use of characteristic function technology.
- Numerical tests show that our model is effective on foreign equity option pricing.

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## ABSTRACT

In this paper we propose a general foreign equity option pricing framework that unifies the vast foreign equity option pricing literature and incorporates the stochastic volatility into foreign equity option pricing. Under our framework, the time-changed Lévy processes are used to model the underlying assets price of foreign equity option and the closed form pricing formula is obtained through the use of characteristic function methodology. Numerical tests indicate that stochastic volatility has a dramatic effect on the foreign equity option prices.



## 1. Introduction

With the growth in globalization of investments and the continued liberalization of cross-border cash flows, the currency translated foreign equity options (cross-currency options) have gained wider popularity. Foreign equity options are contingent claims where the payoff is determined by an equity in one currency but the actual payoff is done in another currency. By a variety of combinations on linking foreign asset price and exchange rate, foreign equity options traded on international markets provide an efficient means of managing multidimensional risks.

Previous studies dealing with the currency translated foreign equity options usually model the dynamics of asset price and exchange rate with Brownian motions, see, for example, Dravid, Richardson, and Sun [1], Ho, Stapleton, and Subrahmanyam [2], Reiner [3], Toft and Reiner [4], and Kwok and Wong [5]. Duan and Wei [6] priced foreign currency and cross-currency options under GARCH model. However, despite the success of the Black–Scholes model based on Brownian motion and normal distribution, two empirical phenomenons cannot be explained by Black–Scholes model: (1) the asymmetric leptokurtic features and (2) the volatility smile. To incorporate the asymmetric leptokurtic features in asset pricing, a variety of models have been proposed, including, among others, fractal Brownian motion, stable processes, Lévy processes, and

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stochastic volatility and GARCH models. For the applications of these alternative models, see, for example, Ivanow et al. [7], Podobnik et al. [8], Podobnik et al. [9], Wang et al. [10,11], Wang et al. [12], Xiao et al. [13,14], Podobnik et al. [15].

Simultaneously, jumps are clearly identifiable from equity data, see, for example, Eraker [16], and references therein. Many studies have been conducted to modify the Black–Scholes model, see, for example, Merton [17], Heston [18], Bakshi, Cao, and Chen [19], Bates [20], Duffie and Singleton [21], Geman, Madan, and Yor [22], Kou [23], Carr and Wu [24], Zhang et al. [25]. In the exchange rate modeling, Brownian motions are also contradicted with empirical phenomenon. Many studies indicate that jumps are important components of the exchange rate dynamics, see, for example, Jorion [26], Johnson and Schneeweis [27], Bates [28,29], Guo and Hung [30], and Carr and Wu [31].

Huang and Hung [32] went beyond the traditional Black–Scholes framework and priced foreign equity options with jumps in foreign asset prices and exchange rate. In Huang and Hung's paper, the exchange rate and foreign asset prices are modeled as multidimensional Lévy processes and the option value is calculated with the Fourier inverse transformation. However, in Ref. [32] model, they assumed the volatility of underlying asset returns of foreign equity option constant. Ref. [33] proposed a new foreign equity option pricing model that allows for the volatility to vary stochastically over time, but without jumps occurring in foreign asset prices and exchange rate processes. The assumption in above both papers differs from many empirical study results that return volatilities vary stochastically over time, and jumps are clearly identifiable from equity and exchange rate data. The purpose of this paper is to explore the use of time-changed Lévy processes as a way to capture these facts, and the closed form pricing formula of foreign equity option is obtained. Following [34], fast Fourier transform of option prices is derived. The foreign equity option pricing model used by Ref. [32] is a special case of our pricing model when the stochastic clock on which the Lévy process is run becomes a calendar time. As in Ref. [24], we can regard the original clock as calendar time and the new random clock as business time. A more active business day implies a faster business clock, and randomness in business activity generates randomness in volatility.

This paper is organized as follows. In Section 2, we introduce the Lévy characteristics and types of Lévy processes. Section 3 presents the fundamental theorem simplifying the calculation of the characteristic function of the time-changed Lévy process. Section 4 shows foreign equity option pricing based on time-changed Lévy process. The concluding remarks are given in Section 5.

## 2. Lévy processes

Lévy processes constitute a wide class of stochastic processes whose sample paths can be continuous, mostly continuous with occasional discontinuities, and purely discontinuous. Generally, Lévy processes are a combination of a linear drift, a Brownian motion, and a jump process. The classic Black–Scholes (BS) model is characterized as the only continuous Lévy model. For a more complete presentation on the topic of Lévy processes see the books Cont and Tankov [35].

### *2.1. Lévy characteristics*

For the remainder of the paper, we fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$ and a standard complete filtration $F = \{\mathscr{F}_t | t \geq 0\}$. The following definition formalizes the class of Lévy processes (see Ref. [35, P. 68]).

**Definition 1** (*Lévy processes*)**.** A right-continuous with left limits stochastic process $(X_t)_{t\geq 0}$ on $(\Omega, \mathscr{F}, \mathbb{P})$ with values in $\mathbb{R}^d$ such that $X_0 = 0$ is called a Lévy process if it possesses the following properties:

1. Independent increments: for every increasing sequence of times $t_0, \ldots, t_n$, the random variables $X_{t_0}, X_{t_1} - X_{t_2}, \ldots, X_{t_n} - X_{t_{n-1}}$ are independent.
2. Stationary increments: the law of $X_{t+h} - X_t$ does not depend on $t$.
3. Stochastic continuity: $\forall t > 0$, $\lim_{h\to 0} \mathbb{P}(|X_{t+h} - X_t| \geq \epsilon) = 0$.

By the Lévy–Itô decomposition, any Lévy process $X_t$ on $\mathbb{R}^d$ can be written as following representation form (see Ref. [35, Proposition 3.7]):

$$X_t = \gamma t + B_t + \int_{|x|\geq 1, s\in[0,t]} xJ_X(\mathrm{d}s \times \mathrm{d}x) + \lim_{\epsilon\downarrow 0} \int_{\epsilon\leq|x|<1, s\in[0,t]} x\{J_X(\mathrm{d}s \times \mathrm{d}x) - \nu(\mathrm{d}x)\mathrm{d}s)\}, \tag{1}$$

where $\gamma \in \mathbb{R}^d$ is a constant vector, $B_t$ is a $d$-dimensional Brownian motion with covariance matrix $A$, and $J_X$ is a Poisson random measure on $[0, \infty) \times \mathbb{R}^d$ with intensity $\nu(\mathrm{d}x)\mathrm{d}t$. In particular, Lévy measure $\nu$ is defined on $\mathbb{R}_0^d$ ($\mathbb{R}^d$ less zero) with

$$\int_{\mathbb{R}_0^d} (1 \wedge x^2)\nu(\mathrm{d}x) < \infty,$$

and describes the arrival rates for jumps of every possible sizes for each component of $X$. The Lévy–Itô decomposition entails that every Lévy process is specified by the vector $\gamma \in \mathbb{R}^d$, the positive semi-definite matrix on $A \in \mathbb{R}^{d\times d}$, and