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Time-singularity multifractal spectrum distribution based on detrended fluctuation analysis

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HIGHLIGHTS

- We propose the multifractal spectrum distribution based on DFA (DFA-MFSD).
- We prove the equivalent of proposed DFA-MFSD and to the standard time-singularity MFSD.
- DFA-MFSD indicates less computational cost and more adaptable than WTMM-MFSD.
- Simulation on real data shows excellent theoretical and practical performances of DFA-MFSD.

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ABSTRACT

The time-singularity multifractal spectrum distribution (TS-MFSD) generalizes the singularity spectrum in a time-varying framework. In this paper, a new method to compute MFSD based on detrended fluctuation analysis (DFA-MFSD) is introduced. We relate DFA-MFSD method to the standard partition function based multifractal spectrum distribution formalism, and prove that both approaches are equivalent for fractal time series with compact support. Furthermore, we find that DFA-MFSD has equivalent results, better mathematic foundation, less computational cost and is more adapted for fractal time series with arbitrary length, compared with MFSD based on wavelet transform modulus maxima (WTMM-MFSD). By analyzing several examples, this paper shows that DFAm-MFSD with different polynomial fitting orders can reliably determine the time-varying multifractal scaling behavior of time series, including processes embodying chirp-type or oscillating singularities. To illustrate these results, simulations are executed using binomial multiplicative cascades, wavelet series and real sea clutter, and simulations indicate that DFA*m*-MFSD benefits from excellent theoretical and practical performances.

1. Motivation

Multifractal theory has become a hot topic in nonlinear science, and has been widely applied in various fields [1] such as electroencephalograms (EEG) [2], electrocardiograms (ECG) [3], turbulent flows [4], seismicities [5], DNA sequences [6], stock market [7], geographical objects [8] and so on. In the above studies, fractal dimensions and multifractal spectrum (MFS) are two important conceptions [9]. MFS is the enhanced conception of overall fractal dimension, which de-

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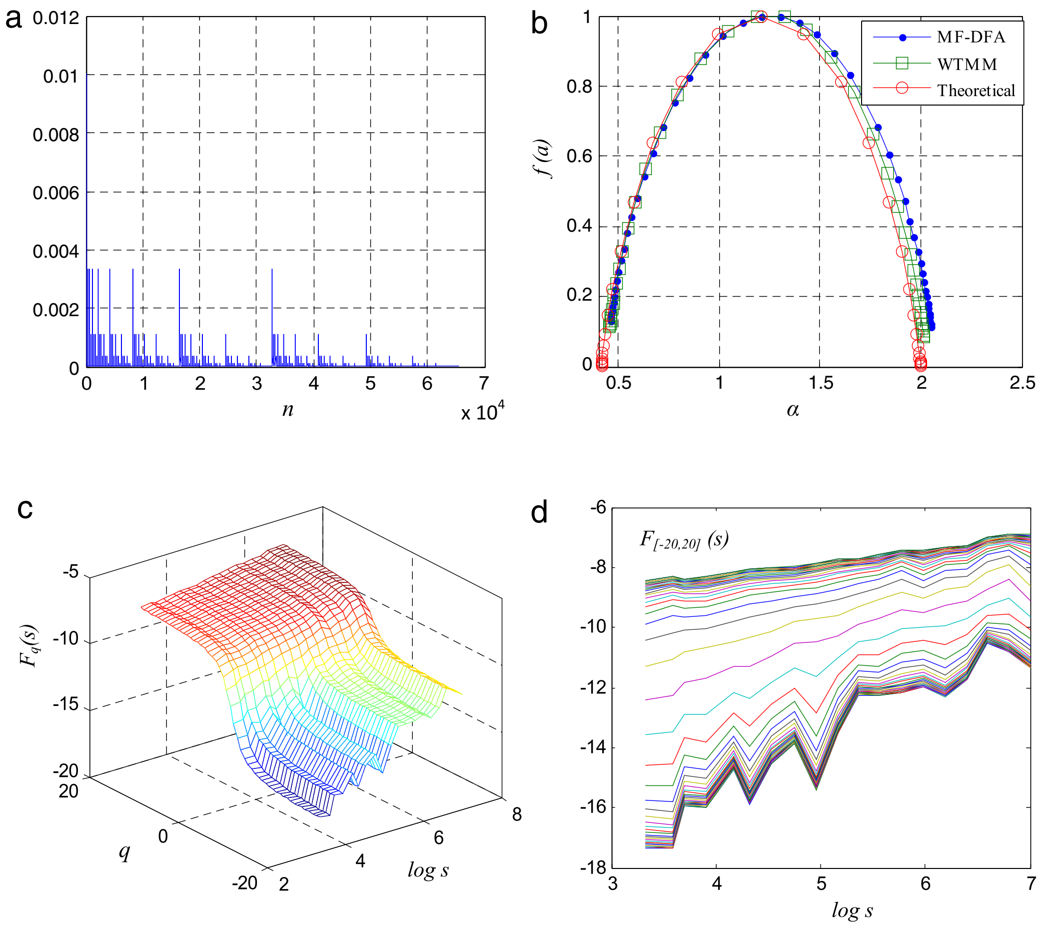

Fig. 1. Traditional multifractal analysis of Binomial Multiplicative Cascades (BMC) based on MF-DFA and WTMM method. (a) BMC with $p_1 = 0.75$, $p_2 = 0.25$, $nbpoints = 2^{16}$, (b) traditional multifractal spectrum, where solid line with isolated points (•), (□) and (○) are MF-DFA3, WTMM and theoretical spectra, respectively. (c) The fluctuation function based on MF-DFA3 with $q \in [-20, 20]$ and (d) the profile of fluctuation function of BMC.

scribes fractal dimensions of fractal subsets by statistical analysis of singularity exponent. There are several algorithms for computing MFS, such as structure function method, wavelet coefficient method [10], wavelet transform model maxima (WTMM) [11], detrending fluctuation analysis(MF-DFA), detrending moving average (MF-DMA) [12,13] and wavelet leaders method (WLMF) [14]. The comparison of the performance of DMA and DFA methods was also investigated concerning the behavior on different scales, different orders of detrending and different types of correlations of the signal [15,16]. Recently, a new multifractal method intended for multiple non-stationary signals has been proposed as a MF version of detrended cross-correlation analysis [17–22]. All these methods mainly focus on estimation accuracy, statistical convergence, effects of limited data length, calculation complexity, stability and the mathematical foundation [23].

However, as signal representation in singularity domain, MFS only adapts to the determined fractal signal or stochastic stationary fractal signal. MFS fails to describe the characteristics of the instantaneous singularity spectrum at any given moment, when the signal, such as complex turbulent, oscillating singularity signal and geophysics signal, possesses space–time non-stationary characteristics and multifractal spectrum is changing over time [24]. In the past years, Wang [24] and we [25] have respectively studied windowed MFS, deduced the definition of short-time singularity analysis, and proposed the algorithm of short-time MFS based on WTMM. Recently, time-singularity *multifractal spectrum distribution based on WTMM* method (WTMM-MFSD) [26] *and multifractal spectrum distribution based on wavelet leaders* (WL-MFSD) [27] have been proposed to determine the time-varying multifractal distribution. Refs. [17,18] proposed *multifractal detrended cross-correlation analysis* (MFDCCA) to investigate the multifractal behaviors in the power-law cross-correlations between two time series or higher-dimensional quantities recorded simultaneously.

Here, we introduce a new algorithm for time-singularity multifractal spectrum distribution (TS-MFSD) based on the multifractal detrended fluctuation analysis, i.e., DFA-MFSD, which is validated to achieve the statistical advantage in computational complexity, precision and calculating convergence. The DFA-MFSD method, which absorbs fully advantage of MF-DFA, does not require the modulus maxima tracking procedure. Hence it involves no more effort in programming than the conventional MF-DFA.