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A permeability model for power-law fluids in fractal porous media composed of arbitrary cross-section capillaries

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HIGHLIGHTS

- A permeability model for power-law fluids in porous media is developed.
- The permeability is a function of fluid property and structure parameters of media.
- The effects of structure parameters on permeability and velocity are discussed.
- The average flow velocity is compared with existing macroscopic model.
- Our permeability model can be considered as an extension of Yu and Cheng's model.

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ABSTRACT

The fractal theory and technology has been applied to determine the flow rate, the average flow velocity, and the effective permeability for the power-law fluid in porous media composed of a number of tortuous capillaries/pores with arbitrary shapes, incorporating the tortuosity characteristic of flow paths. The fractal permeability and average flow velocity expressions are found to be a function of geometrical shape factors of capillaries, material constants, the fractal dimensions, microstructural parameters. The effects of the porosity, the tortuosity fractal dimension, material constants, and geometrical shape factors on the effective permeability are also analyzed in detail. To verify the validity of the present model, our proposed model is compared with the available macroscopic model and experimental data and there is good agreement between them.

1. Introduction

Modeling of flow and transport properties in porous media is an active field of interest for a variety of industrial applications, such as oil reservoirs [1–7], textile industry [8–10], and biological tissues and organs [11,12]. The macroscopic transport properties of porous media such as the flow rate, the average velocity, and the permeability mainly depend on the microstructure of porous media. Since the 1880s, large numbers of literatures have begun to investigate flow properties of porous media. For instance, early as in 1856, the preliminary experimental measurement for Newtonian fluids flow through porous media was undertaken by Darcy [13]. To better understand the mechanisms of hydrodynamics of liquid

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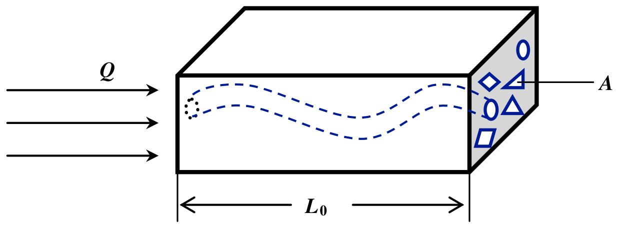

Fig. 1. A sketch of porous media composed of a bundle of tortuous capillaries with various shapes.

flows in smooth wall microchannels (relative roughness <0.5%), Baviere R. et al. [14] performed the experiment on water flow through smooth rectangular microchannels and proved that a friction factor was correctly predicted by Navier–Stokes equation at the small scales. Alexandre lavro [15] studied the power-law fluid flow in a rough fracture of regular or irregular topography by using numerical simulations. Yazdchi K. [16] investigated the macroscopic permeability of fibrous porous media taking its microstructure into account by the finite element method. However, the results obtained from these experimental measurements and numerical simulations are usually correlated as curves or empirical correlations which contained one or several empirical constants, and could not reveal some basic physical mechanisms for flow in porous media.

The capillaries or pores in real porous media are usually non-uniform in size, irregular, and randomly distributed. Fortunately, some published literatures have shown that the fractal geometry may have the potential in analysis of flow and transport properties in porous media [8,17–21]. Yu and Cheng [22] developed the fractal permeability model for bi-dispersed porous media based on the fractal characteristic of porous media. In order to investigate the starting pressure gradient in dual-porosity medium, Wang et al. [23] also applied the fractal theory and technique to obtain an analytical expression of the starting pressure gradient for Bingham fluids in porous media embedded with randomly distributed fractal-like tree networks. Xu et al. [24] developed a probability model for Newtonian fluids radial flow in fractured porous media based on fractal theory and Monte Carlo simulation. Recently, Shou et al. [9] obtained the fractal model for gas diffusion across nanoscale and microscale fibrous media based on fractal theory and capillary model. Luis Guarracino [25] presented a physically-based theoretical model which described the temporal evolution of porosity, saturated and relative permeabilities, retention curve and diffusion coefficient during rock dissolution based on the assumption that rocks consisted of cylinder capillaries with fractal tortuosity and cumulative pore size distribution. However, all the above-mentioned models were developed based on porous media composed of a bunch of parallel cylindrical capillaries. But lately, on the basis of the parallel plate model and the fractal scaling law of length distribution of fractures, a fractal model for permeability for fractured rocks was proposed by Miao et al. [26]. Although Miao's model was in good agreement with numerical simulations, it is not applicable to Non-Newtonian fluids flow through porous media consisting of arbitrary cross-sectional capillaries. In reality, however, most real porous media in the nature is tremendous complex and consists of numerous tortuous capillaries/pores with various shapes. Therefore, it is very meaningful to develop a more general permeability model by considering pores of various geometrical shapes, which can reveal the flow problems of Non-Newtonian fluids in real porous media.

In this article, we aim to develop the fractal permeability model for porous media composed of arbitrary cross sectional capillaries/pores, incorporating tortuous effects for fracture flow of Non-Newtonian power-law fluids. First, the flow rate for Non-Newtonian power-law fluids flow through a single duct of arbitrary cross section is introduced. Then based on the fractal theory, the flow rate, the average flow velocity as well as the fractal permeability model for Non-Newtonian power-law fluids in fractal porous media composed of arbitrary cross-sectional capillaries are derived. Meanwhile, the proposed model is verified by comparing its results with the available macroscopic model. Finally, we also rigorously examine our fractal permeability model, and come to a conclusion that the permeability model of Yu and Cheng [22] can be considered as a special case of our present permeability model.

2. The flow rate for Non-Newtonian power-law fluids in porous media composed of arbitrary shape capillaries

In the following section, some assumptions of the present model for Non-Newtonian power-law fluids flow in porous media are summarized as:

(i) incompressible, full developed, steady state, continuum, and laminar flow;
(ii) negligible surface effect and body forces such as gravity, Coriolis, no slip boundary condition;
(iii) a single duct of constant cross-sectional area A and constant perimeter C;
(iv) all ducts have no branch and no intersection;
(v) power-laws fluids flow through ducts in one direction;
(vi) porous media composed of a bundle of tortuous capillaries with various shapes as illustrated in Fig. 1.