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Vertex-degree sequences in complex networks: New characteristics and applications

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HIGHLIGHTS

- The vertex-degree sequence in a general scale-free network is important for both theory and applications, which is characterized in this article.
- New findings on the major characteristics of vertex-degree sequence in a general scale-free network are reported.
- New findings have great potential applications in complex network modeling, identification and analysis.

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ABSTRACT

Many complex networks exhibit a scale-free vertex-degree distribution in a power-law form $ck^{-\gamma}$, where k is the vertex-degree variable and c and γ are constants. To better understand the mechanism of power-law formation in real-world networks, it is effective to explore and analyze their vertex-degree sequences. We had shown before that, for a scale-free network of size N, if its vertex-degree sequence is $k_1 < k_2 < \cdots < k_l$, where $\{k_1, k_2, \ldots, k_l\}$ is the set of all unequal vertex degrees in the network, and if its power exponent satisfies $\gamma > 1$, then the length l of the vertex-degree sequence is of order $\log N$. In the present paper, we further study complex networks with an exponential vertex-degree distribution and prove that the same conclusion also holds. In addition, we verify our claim by showing many real-world examples. We finally discuss some applications of the new finding in various fields of science and technology.

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1. Introduction

Complex networks are ubiquitous in nature and human society, including such typical examples as the Internet, the World Wide Web, power grids, transportation networks, and biological networks.

Recently, studies in the theory and modeling of complex networks have a renewal of interest in various generic and realistic models, especially the small-world network model [1] and scale-free network model [2]. Scale-free networks, in particular, have an abnormal vertex connectivity, where a small fraction of vertices are highly connected. The scale-free network model of Barabási and Albert [2] revealed an essential power-law distribution of vertex degrees, in the form of $P(k) \propto k^{-\gamma}$ where k is the degree variable and γ is a constant. This power-law distribution is a direct consequence of two

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general mechanisms that generate the network topology: (i) network expansion over time through addition of new vertices; (ii) preferential attachment of new vertex to those existing ones in the network.

In many real-world scale-free networks, the power-law exponent satisfies $\gamma \geq 2$ by default [3–8], but there are also many others with $\gamma < 2$ [9–12]. Therefore, it is important to study the similarities and differences between such networks with $\gamma \geq 2$ and $\gamma < 2$, respectively. In our previous work [7,8,13], we presented some necessary conditions for the scale-free property formation in complex networks, based on the assumption of $\gamma > 1$, as summarized in the Abstract above. This paper further shows that the same conclusion holds also for complex networks with an exponential vertex-degree distribution.

2. Scale-free networks and existing results

A complex network can be represented by an undirected or a directed graph, $G(V, E)$, where V is the set of vertices and E the set of edges. A graph has a number of local and global parameters that characterize its structure (e.g., regularity, modularity), connectivity (e.g., density, diffusion) and robustness (e.g., resilience to random attacks or malicious faults).

The following list summarizes the main parameters to be used in this paper:

- M Number of edges; $M = |E|$
- N Number of vertices; $N = |V|$
- k_i Degree of vertex $i \in V$
- $\bar{d}$ Average vertex degree of the network; $\bar{d} = 2M/N = \sum_{i\in V} k_i/N$
- n_{k_i} Number of degree-k_i vertices: $\sum_{i=1}^{l} n_{k_i} = N$ and $\sum_{i=1}^{l} n_{k_i} k_i = 2M$
- l Length of all unequal vertex-degree sequence $\{k_1, k_2 \ldots, k_l\}$, $1 \leq k_1 < k_2 \ldots < k_l$
- K Set of all vertex degrees in the network
- $P(k)$ Degree distribution, or fraction of vertices of degree k : $P(k) = n_k/N$.

For scale-free networks, one has

$$P(k) = ck^{-\gamma}, \quad \gamma > 1.$$

Here, the requirement of $\gamma > 1$ ensures that $P(k)$ can be normalized. The constant c is used for normalization, $c = \left(\sum_{k\in K} k^{-\gamma}\right)^{-1}$.

In our previous works [7,8], we proved that for a scale-free network of size N, having a power-law distribution with exponent $\gamma \geq 2$, the number of degree-1 vertices, if not zero, tends to be of order N; and we also proved that the average degree is of order lower than $\log N$. Our method provides an analytical tool that helps one to check if a given network is scale-free, which relies on static conditions that can be easily verified. Furthermore, we showed that the number of degree-1 vertices is divisible by the least common multiples of $k_1^{\gamma}, k_2^{\gamma}, \ldots, k_l^{\gamma}, \gamma \geq 1$, if they are all integers, where $k_1 < k_2 < \cdots < k_l$ is the vertex-degree sequence of the network. This leads a remodeling method to equip a scale-free network with some small-world features. Based on our results [7,8], we further showed [13] that for scale-free networks with $\gamma > 1$, the length l of the vertex-degree sequence is of order $\log N$. Here, we must emphasize that this result is very important, which demonstrates that the length of the degree sequence is an essence of a general scale-free network. In fact, all scale-free networks have very small-numbered degree sequences comparing with the network size values. Utilizing this characteristic, one can reconstruct a scale-free network with prominent small-world features [8] and can also improve some commonly-used maximal-degree search algorithms.

We must also stress that the above conclusion holds based on the precondition that the network obeys a precise power-law vertex-degree distribution. Actually, many real-world scale-free networks are not exactly so, which only have approximate scale-free features, so there are subtle differences between such real networks and our theoretical results.

Furthermore, we must point out that for many real-world networks, the lengths of their degree sequences are at most of order $(\log N)^{\varepsilon}$, namely $l \leq O((\log N)^{\varepsilon})$, where ε is a very small constant. This means that l is very small as compared with the network size value. We will verify this by some real networks in different fields near the end of this paper.

3. A new characteristic of scale-free networks and its derivation

3.1. Vertex-degree sequences in scale-free networks

In Section 3, we present a new property of scale-free networks and its mathematical derivation.

Suppose that the vertex-degree sequence of network is $1 \leq k_1 < k_2 < \cdots < k_l$. For scale-free networks, one has [3]

$$P(k_i) = \frac{n_{k_i}}{N} = ck_i^{-\gamma}. \tag{1}$$

Here, n_{k_i} is the number of vertices with degree k_i, satisfying

$$N = \sum_{i=1}^{l} n_{k_i}. \tag{2}$$

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