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GARCH models, tail indexes and error distributions: An empirical investigation☆

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A B S T R A C T

We perform a large simulation study to examine the extent to which various generalized autoregressive conditional heteroskedasticity (GARCH) models capture extreme events in stock market returns. We estimate Hill's tail indexes for individual S&P 500 stock market returns and compare these to the tail indexes produced by simulating GARCH models. Our results suggest that actual and simulated values differ greatly for GARCH models with normal conditional distributions, which underestimate the tail risk. By contrast, the GARCH models with Student's *t* conditional distributions capture the tail shape more accurately, with GARCH and GJR-GARCH being the top performers.

1. Introduction

The financial crisis has reminded us that the quality of statistical models for risk management is often lower than expected (Daníelsson, 2008). While the models typically work well for small shocks, they often fail in crisis times characterized by extreme events. Therefore, the adoption of the appropriate risk management model to assess the expected financial losses in stock markets remains a challenge (Rossignolo, Fethib, & Shaban, 2012).

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Generalized autoregressive conditional heteroskedasticity (GARCH) models have become the most popular models (Engle, 2001) of the conditional variance of stock returns for many purposes, ranging from portfolio optimization, day-to-day risk management and margin calculation to regulatory reporting under the Basel framework. Despite many variations, the application of simple Gaussian GARCH models is most common (Hansen & Lunde, 1997). These models are established to successfully capture key stylized facts about stock returns volatility clustering and fat-tailed return distributions.

The aim of this paper is to examine precisely how different GARCH models are able to capture/model the tail behavior of various equity stock prices using extreme value theory (EVT) as a basis for our simulation study. Correct modeling of tail behavior is key to properly managing risks (e.g., calculating capital requirements), optimizing portfolios, designing stress testing scenarios and generally improving understandings of stock market dynamics. A related discussion on the assessment of the unconditional distributions of financial time series using EVT is available in Daníelsson and de Vries (1997b, 2000), Daníelsson, Hartmann, and de Vries (1998), Embrechts, Resnick, and Samorodnitsky (1998) and Longin (2000).

We analyze whether there is a particular GARCH model that outperforms other GARCH models in terms of correctly assessing the shape of the tail distributions. Underestimation of fat tails in the loss distribution leads to systematic undervaluation of the risk hidden in stock returns. In fact, the increased Value at Risk (VaR)[1] buffers imposed by Basel III are the result of undervaluation of the fatness of the tails of the loss distributions (Basel, 2007, 2011). The importance of tail fatness for capital reporting and Value at Risk calculations is emphasized in Huisman, Koedijk, and Pownall (1998) and, more recently, for VaR estimation using EVT in Karmakar (2013) and Sun and Zhou (2014).

In this study, we quantify the magnitude with which various GARCH models capture and reproduce the tail fatness of the unconditional loss distribution based on a large data set. The analysis starts by assessing the tail behavior of all series by calculating the tail indexes using the Hill method (Hill, 1975) modified by Huisman, Koedijk, Kool, and Palm (2001). The tail index is a characteristic of the tail behavior of a given distribution. For example, in case of the Student's *t* distribution, its reciprocal coincides with the degrees of freedom; intuitively, the smaller the value, the lighter tails of the distribution. Specifically, depending on the value of the tail index, the distribution has one of the following characteristics a short tail with a finite terminal value, a light tail with no terminal value, or a fat tail with no terminal value that slowly approaches infinity. Needless to say, asymmetric distributions may have different tail indexes for each tail. We focus on the minima of the returns or, in other words, on the maxima of the loss distributions.

We estimate 8 different GARCH-family models (with various distributional assumptions and lag structures) for stocks currently listed on the S&P 500 stock market index and estimate tail indexes for the individual series of this stock market index. Thus, we perform Monte Carlo simulations of all the models to replicate the return series. For each simulated series, we calculate the tail index, and thus, we assess the model-implied tail index. Consequently, we are able to compare the tail behavior of the actual time series to the tail behavior implied by the model. We motivate our analysis by the fact that there is a non-trivial analytical expression to calculate the model-implied tail index for a simple GARCH(1,1) model (Groenendijk, Lucas, and de Vries (1995), and the methodology further extended by Sun and Zhou (2014)), but an analytical solution does not exist for more complicated specifications of GARCH models. In this paper, we present a procedure, which allows us to compare various GARCH models including those, for which Hill's α cannot be calculated analytically. Therefore, we perform a Monte Carlo simulation study of tail indexes. A similar analysis has been conducted by Mikosch and Stãricã (2000), who find that, although GARCH-family models generally reproduce fat-tailed return series, the tails captured by some models are lighter then the actual data show. By contrast, our paper employs different methods to evaluate the tail shape of GARCH models with longer lag structure, and we use a large data set accompanied by an extensive simulation study. Sun and Zhou (2014) show how to calculate tail index for (G)ARCH models with (1,1) lag structure analytically for both the normal

[1] VaR is a quantile based measure used for regulatory reporting purposes, day-to-day risk management, trading desk limit setting, etc.