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Optimal insurance contract under VaR and CVaR constraints

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ABSTRACT

This study endogenously develops an optimal insurance contractual form for maximizing insured expected utility under VaR and CVaR constraints. We find that CVaR constraint does not affect the contractual form, but may increase minimum insurance premium requirement. Additionally, when the VaR constraint is binding, the optimal contract is a double deductible insurance. However, if the contract is restricted to a regular form (both indemnity schedule and retained loss schedule are continuously nondecreasing) for avoiding moral hazard problem, the optimal contract is a piecewise linear deductible insurance. Finally, we provide intuitive comparison between this study result and relevant studies.

1. Introduction

The basic principles of the 2001 proposal of the Basel Banking Supervisory Committee suggested that VaR (value-at-risk) is a key measure of risk (Szegö, 2002).[2] VaR has become a popular measure

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[2] VaR is a widely applied measure of risk used in financial risk management. VaR is defined as the worst expected loss over a given horizon and confidence level, such as 95 or 99 percent (Jorion, 2009).

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of downside risk, and is widely used by financial institutions to monitor their exposure, and by regulators to set capital margins. However, VaR is merely a particular percentage quantile of losses while CVaR equals the average of the worst losses that exceed or equal VaR.[3] Thus, CVaR is intuitively thus superior to VaR as a risk measure for risk management purposes. Additionally, unlike VaR, CVaR is a coherent measure of risk.[4] The coherency of CVaR contributes to the convenience of its application in mathematical treatment, risk management and portfolio selection. Naturally, the insured (including individuals, banks and other institutions) and insurers can purchase insurance and reinsurance for risk reduction. Since VaR and CVaR are prevalent criteria of legislative risk management, this study considers optimal insurance imposed with the two risk constraints. Previous investigations (Huang, 2006; Wang, Shyu, & Huang, 2005; Zhou and Wu, 2009) have examined the optimal insurance with constraints on VaR. Thus this study primarily optimizes insurance form endogenously under constraints regarding the CVaR of the insured or the insurer. Because the CVaR value is calculated from VaR, a CVaR constraint frequently accompanies the VaR constraint. This study thus considers the optimal insurance contract which meets both the VaR and CVaR constraints.

For the risk-averse insured, Raviv (1979) demonstrated that Pareto optimal insurance is *straight deductible insurance* for a risk neutral insurer; otherwise, optimal insurance involves coinsurance of losses above the deductible if the insurer is risk averse. Raviv (1979) devised a standard approach of endogenously developing an optimal insurance form. Subsequent investigations are presented as follows. Gollier (1996) showed that the optimal insurance contract contains a "disappearing deductible" for a prudent insurer ($U''' > 0$), if the risk on the uninsurable asset increases with the size of the insurable asset loss. Additionally, if background wealth (uninsurable risk) is conditionally normally distributed given insurable loss, Huang, Shiu, and Wang (2013) demonstrated that the optimal insurance contract may be proportional coinsurance above a straight deductible for a quadratic, negative exponential, or mean-variance utility function. Moreover, Wang and Huang (2012) designed an optimal insurance contact for an insured party with loss aversion utility preference. The representative optimal insurance form is *truncated deductible insurance*, where the losses exceeding a critical level are retained, and otherwise a deductible insurance is adopted.

Due to legislative restrictions or risk management requirements, several studies have incorporated risk measure tools (such as VaR and CVaR) into optimal insurance contracts. These studies mostly belong to either of the following two categories. The first category maximizes the objective of the insured under some risk constraints. For example, Wang et al. (2005) designed an optimal insurance contract in which the insured maximizes the expected final wealth under a VaR constraint. Building on Wang et al. (2005), Huang (2006) designed an insurance contract under a VaR constraint, where the insured is risk averse and faces a VaR constraint. Unlike Huang (2006), Zhou and Wu (2009) developed the optimal insurance under the VaR constraint of the insurer. Additionally, Zhou and Wu (2008) derived the optimal insurance under the insurer risk constraint, where the risk constraint is that the expected loss of terminal wealth suffered by the insurer is maintained below some prespecified level. Like Zhou and Wu (2008), Zhou, Wu, and Wu (2010) derived the optimal insurance in the presence of loss limit of the insurer, where the loss limit means the insurer wishes to limit the loss after the indemnity payments below some prespecified level. The main optimal contractual forms are deductible, *double deductible* and *limited deductible insurances* in the models of Wang et al. (2005), Huang (2006) and Zhou et al. (2010), respectively. Additionally, the optimal contract in Zhou and Wu (2008, 2009) has two forms, deductible insurance and *piecewise linear deductible insurance*.[5]

Next, the second category minimizes the extent of risk measurement. The category mainly includes the optimal reinsurance studies with respect to the minimum VaR and CVaR of the insurer loss. Thus, the corresponding optimization problem is called the VaR- and CVaR-optimization. For instance, under the constraint that the indemnity function is nondecreasing convex, the optimal insurance in Cai,

[3] The formal definition of CVaR will be presented in Section 2.

[4] A coherent risk measure is a function that satisfies four properties: monotonicity, sub-additivity, homogeneity, and translation invariance. However, VaR merely satisfies three properties and does not respect the sub-additivity property (Artzner, Delbaen, Eber, & Heath, 1999).

[5] The definitions of double deductible insurance, limited deductible insurance and piecewise linear deductible insurance are shown in Table 1 of Section 2.