North American Journal of Economics and Finance 37 (2016) 128–147

Contents lists available at ScienceDirect

North American Journal of Economics and Finance

journal homepage: www.elsevier.com/locate/ecofin

Convergence rates of recombining trees for pricing options on stocks under GBM and regime-switching models with known cash dividends

Jingtang Ma*, Jiacheng Fan

School of Economic Mathematics, Southwestern University of Finance and Economics, Chengdu 611130, PR China

ARTICLE INFO

Article history:
Received 5 December 2015
Received in revised form 30 March 2016
Accepted 31 March 2016
Available online 13 April 2016

Keywords:
Option pricing
Known cash dividends
Binomial trees
Convergence rates
Regime-switching models

ABSTRACT

In the literature there appear various kinds of binomial trees for pricing options on stocks under geometric Brownian motions (GBMs) with known cash dividends. The aim of this paper is to compare the performance of the existing binomial trees in aspect of the convergence rates, which are usually used to measure precisely how fast the approximate values converge to the exact one, and to give a theoretical proof of the convergence rates for the interpolation binomial trees which are based on a model that excludes the arbitrage possibilities. Also the paper extends the studies to the regime-switching models with known cash dividend payment.

1. Introduction

If a continuously paid dividend yield is used or the future dividend is specified as a fixed percentage of the stock price at dividend dates, then the classical option pricing models of Merton (1973), Black and Scholes (1973) can be used with only some minor modifications; however in practice a fixed cash value of dividend (instead of percentage) is often paid discretely in time and this causes challenging in the option pricing.

* Corresponding author. Tel.: +86 28 87092956.
E-mail addresses: mjt@swufe.edu.cn (J. Ma), 41226011@2012.swufe.edu.cn (J. Fan).

http://dx.doi.org/10.1016/j.najef.2016.03.011

In the literature, there are three basic models for the option pricing when the underlying stock pays fixed cash dividends in the future, namely, escrowed model, forward model and piecewise lognormal model (see for example the overview by Frishling (2002) or Vellekoop and Nieuwenhuis (2006)). Denote the stock price by S_t, the dividend payable at time t_i by D_{t_i}, the maturity date by T, the interest rate by r, and the volatility by σ.

1.1. Escrowed model

Assume that the stock price minus the present value of all dividends to be paid until the maturity of the option follows a geometric Brownian motion. More precisely, the model for the price and capital price will be:

$$dC_t = rC_t dt + \sigma C_t dW_t, \quad S_t = C_t + \sum_{t<t_i<T} D_{t_i} e^{-r(t_i - t)}, \quad S_T = C_T,$$

where W_t is the standard Brownian motion and C_t the capital process (see also Roll (1977), Geske (1979) and Whaley (1981)).

1.2. Forward model

Assume that the stock price plus the future value of all dividends (from past dividend dates to today) follows a geometric Brownian motion. That is

$$dA_t = rA_t dt + \sigma A_t dW_t, \quad S_t = A_t - \sum_{0<t_i<t} D_{t_i} e^{r(t_i - t)}, \quad S_0 = A_0.$$

(see also Heath and Jarrow (1988)).

1.3. Piecewise lognormal model

Assume that the stock price jumps downward at dividend dates with jump size equal to the cash dividend payments at those dates and follows a geometric Brownian motion in between those dividend dates. That is

$$dS_t = rS_t dt + \sigma S_t dW_t, \quad t_i < t < t_{i+1}, \quad S_{t_i}^{+} = S_{t_i}^{-} - D_{t_i}, \tag{1}$$

where $S_{t_i}^{+}$ and $S_{t_i}^{-}$ are stock prices immediately before and after the dividend respectively.

In the escrowed model and forward model, the price of European options can be calculated explicitly using the Black–Scholes' formula with an adjusted values of the current stock price or strike price (see Frishling (2002)). But there are serious problems in these two models. As the stock price does not follow the geometric Brownian motion anymore in these two models, there is mispricing risk for the options with known cash dividend (see for example Frishling (2002)). More seriously this escrowed model admits arbitrage opportunity as discussed by Haug, Haug, and Lewis (2003) (also the paper discusses the stock dependent dividend). As argued by Haug et al. (2003), the piecewise lognormal model can exclude the arbitrage opportunity.

In the literature there are modifications of the escrowed or forward models – mixed escrowed and forward model and volatility adjustment model.

1.4. Mixed Escrowed and forward model

Bos and Vandermark (2002) propose to split the dividend linearly and then reduce the stock price by one part of the dividend and raise the strike price by the other part of the dividend.