North American Journal of Economics and Finance 37 (2016) 217–235

Contents lists available at ScienceDirect

North American Journal of Economics and Finance

journal homepage: www.elsevier.com/locate/ecofin

The extension from independence to dependence between jump frequency and jump size in Markov-modulated jump diffusion models

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A R T I C L E I N F O

Article history:
Received 8 July 2015
Received in revised form 15 April 2016
Accepted 18 April 2016
Available online 10 May 2016

Keywords:
Markov-modulated jump models
EM-gradient algorithm
SEM algorithm

A B S T R A C T

We set out in this study to investigate the relationship between jump frequency and jump size for the 30 component stocks of the Dow Jones Industrial Average (DJIA) index, extending the Markov-modulated jump diffusion model from independence to dependence between jump frequency and jump size. We propose an estimation method for the parameters of the Markov-modulated jump diffusion model based upon dependence between jump frequency and size, with our results indicating that when abnormal events occur, the Markov-modulated jump diffusion models with both state-independent jump sizes (MJMI) and state-dependent jump sizes (MJMD) outperform the pure jump diffusion (JD) model in terms of capturing the risks of jump frequency and jump size. Based upon Akaike Information Criterion (AIC) and Schwarz Bayesian Criterion (SBC), our results further indicate that for 23 of the component stocks, the MJMD model may be better suited, as compared to the MJMI model. Finally, our empirical observations reveal that the behavior of jump risks in the stock markets, including jump frequency and jump size, is not independent, since

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http://dx.doi.org/10.1016/j.najef.2016.04.003

these phenomena are found to coincide during both financial crisis periods and stock market crashes, with the largest jump size risks, during certain periods, being accompanied by either systematic or idiosyncratic risks.

1. Introduction

Having identified the phenomenon of jumps in the financial markets, many of the prior related studies have gone on to apply the widely-used Merton (1976) jump diffusion models in their analyses.[1] Within the Merton jump diffusion model framework, the fluctuations in stock returns are divided into two elements, the first of which describes the normal fluctuations in stock returns (which are modeled based upon normal distribution), while the second describes the abnormal fluctuations in stock returns with the arrival of abnormal information (good or bad news). Merton (1976) also went on to identify two random factors with compound processes to describe the abnormal fluctuations, namely jump frequency and jump size, where jump frequency is the arrival rate of the information and jump size is the change in the stock return on the occurrence of such abnormal events.

Markov-modulated models have been extensively evaluated in many of the more recent studies[2]; for example, in the application of the asset pricing of Markov-modulated models, Elliott, Siu, Chan, and Lau (2007) investigated the option price under a generalized Markov-modulated jump diffusion model, while Bo, Wang, and Yang (2010) similarly derived the valuation of currency options when the spot foreign exchange rates followed a Markov-modulated jump diffusion model. Elliott and Siu (2013) subsequently provided the hidden Markov chain modulating pure jump asset pricing model, which has proven to be capable of capturing the asymmetric jump features on changes in the stock returns. In their application of 'ruin and default' theory, Huang, Lin, and Wu (2009) proposed a Markov-modulated compound Poisson processes to describe the default behavior of firms.

Chang, Fuh, and Lin (2013) noted that the arrival rate of jump frequency in the Dow Jones Industrial Average (DJIA) index and its component stocks was not constant, and thus, was potentially dependent upon jump risks. They developed a Markov-modulated jump diffusion model in which hidden states were adopted to describe the arrival rates with changes in the level of intensity of the abnormal events. They also provided closed-form solutions for the Markov-modulated jump diffusion model under a general equilibrium framework for option prices, clearly showing that the resultant distribution had certain empirically-observed financial features in the DJIA and its component stocks, including leptokurtic returns, volatility smile and volatility clustering.

Based upon their empirical studies on ten years of stock return data, superior empirical fit over competing Poisson style models was identified by Chang et al. (2013) using the expectation–maximization (EM) algorithm (Dempster, Laird, & Rubin, 1977). The EM algorithm is an iterative method for the 'maximum likelihood estimation' (MLE) of the parameters in the statistical model, comprising of two steps which are referred to as the 'expectation step' (E-step) and the 'maximization step' (M step). The E-step creates a function for the expectation of the completed data log-likelihood which is then evaluated using the current estimates of the parameters in incomplete data, including jump frequency and jump size, with the states being the hidden variables in the Markov-modulated jump diffusion model. The M-step subsequently computes the parameters maximizing the expected log-likelihood previously identified in the E-step.

In the present study, we extend the Markov-modulated jump diffusion model with state-independent jump size, as described in Chang et al. (2013), to the Markov-modulated jump diffusion

[1] See, for example, Ball and Torous (1983), Ball and Torous (1985), Jarrow and Rosenfeld (1984), Feinstone (1987), Akgiray and Booth (1988) and Kou (2002).

[2] Examples include Elliott, Siu, Chan, and Lau (2007), Huang, Lin, and Wu (2009), Bo, Wang, and Yang (2010), Chang, Fuh, and Lin (2013) and Elliott and Siu (2013)