North American Journal of Economics and Finance 37 (2016) 267–278

Contents lists available at ScienceDirect

North American Journal of Economics and Finance

journal homepage: www.elsevier.com/locate/ecofin

Pricing chained dynamic fund protection

Heejae Han, Junkee Jeon*, Myungjoo Kang

Department of Mathematical Sciences, Seoul National University, Seoul 151-747, Republic of Korea

ARTICLE INFO

Article history:
Received 28 January 2016
Received in revised form 10 May 2016
Accepted 11 May 2016
Available online 25 May 2016

Keywords:
Dynamic fund protection
Chained option
Reflection principle
Hitting time

ABSTRACT

In this paper, we propose a new kind of dynamic fund protection (DFP). In contrast to the usual DFP, our newly developed DFP has two protection levels and protection is activated only when the value of underlying asset reaches upper protection levels. This kind of product has a structure similar to that of the chained barrier option proposed by Jun and Ku (2012). In this context, we name our newly designed equity-linked product as chained dynamic fund protection (CDFP). Buying CDFP can be advantageous for both vendors and investors compared to buy usual DFP. Relatively small downside risk for CDFP is beneficial for vendors. Also for investors, the price they cost for protection of CDFP is cheaper than that of usual DFP. Furthermore, investors can handle the price of protection by adjusting the level of upper protection as they desired. In this paper, we derive a closed-form formula for CDFP using reflection principle under Black–Scholes framework. Furthermore, we represent numerical results for values of CDFP according to various parameters.

1. Introduction

An equity-indexed annuity (EIA) is an insurance contract whose earning rate is closely related to an equity index. It offers a minimum guaranteed return in the event of a low return on the stock market. Therefore, the earning rate for an EIA is determined by the maximum of the minimum guaranteed return and the earning rate of the stock market. In general, investing directly in the stock market or in mutual funds involves high risk, while investing in a bond fund or fixed deposit usually returns a

* Corresponding author. Tel.: +82 2 880 4443.
E-mail address: hollman@snu.ac.kr (J. Jeon).

http://dx.doi.org/10.1016/j.najef.2016.05.004
1062-9408/

very low earning rate. EIAs were first developed to overcome such shortcomings of traditional derivatives. They have had great appeal to moderately conservative investors who usually hesitate to take high risks but want a moderate return, and they are considered to be the most innovative financial derivatives in the American insurance industry in the past decade. Since an EIA offers investors a minimum guaranteed return at maturity, it has a more complex structure than other financial derivatives. Especially, option pricing theory can be useful for developing an EIA since EIAs have an implicit option structure. The interested reader can refer to Mitchell and Slater (1996),ee (2003), and Tiong (2000) for a more detailed explanation of EIAs.

In 2000, Gerber and Pafumi (2000) and Gerber and Shiu (2003) proposed the concept of dynamic guarantee or protection as an application of EIAs. Dynamic fund protection (DFP) can be seen as an extended put option in that the seller provides continuous protection to ensure that the balance in the customer's account does not fall below the predetermined protection level. If the fund price falls below the protection level, the vendor immediately upgrades the fund so that the price recovers to a protected level.

There has been substantial research on DFP. Imai and Boyle (2001) studied whether DFP with underlying assets obeys the constant elasticity of variance (CEV) model. Gerber and Shiu (2003) explained the relationship between DFPs and lookback options. Wong and Chan (2007) used an asymptotic method to determine the value of DFP when the underlying asset satisfies a stochastic volatility model. Chu and Kwok (2004) derived pricing formulas for DFP having automatic reset or withdrawal rights. Ko, Shiu, and Wei (2010) conducted research on DFP with a withdrawal benefit.

For investors, DFP can be beneficial in that it protects downside risk. However, in the view of vendors, this means that they must settle for downside risk that they originally did not face. We propose a new kind of EIA that can help to reduce such an unsatisfying risk for vendors. In contrast to the usual DFP, the new EIA activates protection only if the value of the underlying asset reaches a specific upper protection value, which is larger than the initial underlying asset price. In Jun and Ku (2012), they proposed an option with two barrier levels that is activated only when the value of the option reaches a specific barrier level. They called it a chained barrier option and derived a pricing formula for it. Our new DFP has a similar structure with a chained barrier option as well. Jun and Ku (2013) extended the chained barrier option when the barrier is of exponential form, and approximated the value of the American chained barrier option probabilistically in Jun and Ku (2015).

In this context, we name our newly designed EIA chained DFP (CDFP). CDFP has a structure similar to that of chained barrier options, but differs in that its payoff function consists of lookback state variables. CDFP can be a win-win strategy for both vendors and investors. For vendors, CDFP can be beneficial compared to a usual DFP because it reduces the downside risk taken by vendors. In addition, it can be beneficial for investors in that it provides less expensive protection than a usual DFP.

This paper consists of as follows. In Section 2, we review the basic concepts of dynamic fund protection and summarize pricing formulas. In Section 3, we represent the concept of CDFP, a DFP whose protection is activated only if the value of underlying asset reaches predefined upper protection line. We also derive pricing formula for CDFP by utilizing reflection principle of Brownian motion and Girsanov theorem. In Section 4, we present some numerical results for the value of CDFP according to the change of various parameters.

2. Some preliminary results

The usual assumptions of the Black–Scholes option pricing framework are adopted in this paper. Let S_t denote the value of the underlying fund under a risk-neutral measure $\mathbb{P}$. We assume that the underlying fund price follows a geometric Brownian motion:

$$dS_t = rS_t dt + \sigma S_t dW_t \tag{2.1}$$

where $r > 0$ is the risk-free rate of interest and σ is the constant volatility of S_t. W_t is a one-dimensional standard Brownian motion process on a filtered probability space $(\Omega, \mathcal{F}_{t\geqslant 0}, \mathbb{P})$, where $\mathcal{F}_{t\geqslant 0} \equiv \mathbb{F}$ is the natural filtration generated by $(W_t)_{t\geq 0}$.

DFP guarantees a predetermined protection level D to an investor owning the underlying fund. The dynamic protection replaces the original value of the underlying fund by an upgraded value F_t such