North American Journal of Economics and Finance 37 (2016) 393–405

Contents lists available at ScienceDirect

North American Journal of Economics and Finance

journal homepage: www.elsevier.com/locate/ecofin

Portfolio selection with a systematic skewness constraint

Chonghui Jiang [a,b], Yongkai Ma [b], Yunbi An [c,*]

[a] School of Finance, Jiangxi University of Finance and Economics, Nanchang 330013, China
[b] School of Management and Economics, University of Electronic Science and Technology of China, Chengdu 610054, China
[c] Odette School of Business, University of Windsor, Windsor, Ontario, Canada N9B 3P4

ARTICLE INFO

Article history:
Received 20 June 2015
Received in revised form 22 March 2016
Accepted 23 March 2016
Available online 2 April 2016

Keywords:
Portfolio selection
Mean-variance model
Systematic skewness

ABSTRACT

This paper investigates portfolio selection within a mean-variance-systematic skewness framework. We derive the composition of efficient portfolios in our model, and analyze the properties of these efficient portfolios. We show that the required systematic skewness is achieved at the expense of traditional mean-variance efficiency, and that a more stringent systematic skewness constraint induces a greater loss in mean-variance efficiency. Our numerical analysis demonstrates that the presence of the systematic skewness constraint helps improve the skewness of efficient portfolios in our model over the skewness of traditional efficient portfolios.

1. Introduction

There is considerable evidence that asset return distributions are typically asymmetric, with heavier tails than allowed in the normal distribution (Harvey, Liechty, Liechty, & Muller, 2010; Markowitz & Usmen, 1996; Premaratne & Tay, 2002). Moreover, assets tend to be more correlated when the market is down than when the market is up (Ang & Chen, 2002), a phenomenon referred to as asymmetric correlations in asset returns. Thus, portfolios based on Markowitz's (1952) mean-variance theory tend to underestimate a portfolio's tail risk (Agarwal & Naik, 2004; Sortino & Forsey,

* Corresponding author. Tel.: +1 519 253 3000 3133.
E-mail addresses: jch0920@163.com, jiangchonghui@uestc.edu.cn (C. Jiang), mayongkai@uestc.edu.cn (Y. Ma), yunbi@uwindsor.ca (Y. An).

http://dx.doi.org/10.1016/j.najef.2016.03.008
1062-9408/

1996; Sortino & Price, 1994), as Markowitz assumes that only the first two moments of asset returns matter. In particular, Markowitz's portfolios may experience a large opportunity loss during a market downturn, given the fact that financial assets are highly correlated in this circumstance. Therefore, it is important for investors and fund managers to take into account higher-order moments of asset return distributions in portfolio selection in order to enhance portfolio performance.

Previous work in this area documents the influence of both absolute and conditional higher moments on portfolio decisions (Athayde & Flôres, 2004; Harvey et al., 2010; Jondeau & Rockinger, 2003, 2006). The literature rests on the assumption that investors have a preference for odd moments and an aversion to even moments.[1] This assumption reflects the fact that investors prefer a high chance of extreme returns in the positive direction rather than the other way around, and they are averse to the uncertainty of returns. For example, Athayde and Flôres (2004) consider a three-moment model in which variance is minimized subject to skewness and mean return, and characterize the geometric properties of the optimal surface. While higher-moment models are able to produce portfolios that exhibit more desirable risk-return characteristics than does the traditional mean-variance model, incorporating higher-order moments into portfolio selection greatly complicates the optimization process, rendering it hard to solve.

This paper investigates portfolio selection within a mean-variance-systematic skewness framework.[2] In particular, we examine the impact of the systematic skewness constraint on the mean-variance efficiency and skewness of the mean-variance-systematic skewness efficient portfolios in our model. It is noteworthy that an additional skewness constraint can be imposed in the mean-variance portfolio selection problem in order to improve the skewness of the resulting portfolio. The challenge is that such a mean-variance-skewness problem is not analytically tractable, as total skewness is non-linear in portfolio weights. Moreover, for some required skewness and return, the problem may have no solution at all, suggesting that the required skewness cannot be achieved. Motivated by analytical tractability, in this paper we consider a systematic skewness (or coskewness) constraint, rather than the total skewness constraint. Given the fact that systematic skewness is linear in portfolio weights, our mean-variance-systematic skewness problem remains a quadratic programming with a convex objective function, and thereby is analytically tractable. As long as the number of risky assets is larger than or equal to the number of constraints, there is a unique solution to the problem (Karush, 1939; Kuhn & Tucker, 1951). This enables us to characterize the components of the efficient portfolios, and also explore the properties of efficient portfolios determined by our model in order to gain insights into investors' investment behavior.

Our research complements previous literature on portfolio selection with higher-order moments. Similar to higher-moment portfolio selection models, our model is also based on the notion that investors have a preference for skewness. An asset's coskewness measures the contribution of the asset to the skewness of the market portfolio (Moreno & Rodríguez, 2009). Assets with positive coskewness are preferred by investors, as these assets exhibit desirable risk-return characteristics compared with the market portfolio. Imposing the systematic skewness constraint in portfolio selection is able to improve the skewness of the resultant portfolio's return distribution. This is because the presence of the systematic skewness constraint helps produce portfolios that are overweighted with positive coskewness assets and underweighted with negative coskewness assets, compared with the corresponding mean-variance efficient portfolios.

We find that the efficient portfolios in our model consist of the risk-free asset, the tangency portfolio determined by the traditional mean-variance model, and a portfolio constructed to hedge against shocks to market volatility (hedge portfolio). In our model, the required systematic skewness is achieved at the expense of mean-variance efficiency, and a more stringent systematic skewness constraint induces a greater loss in mean-variance efficiency. Using a numerical example, we show that the systematic skewness constraint helps enhance the skewness of efficient portfolios determined by our model over the skewness of traditional mean-variance efficient portfolios.

---

[1] Some recent work shows that investors may be willing to trade low expected returns or high volatilities for high skewness or low kurtosis (Chunhachinda, Dandapani, Hamid, & Prakash, 1997; Dittmar, 2002; Mitton & Vorkink, 2007).

[2] An asset's systematic skewness is the normalized measure of the asset's coskewness, which represents the co-movement between the asset return and the squared market return.