Available online at www.sciencedirect.com

ScienceDirect

Physica A 387 (2008) 3399–3409

PHYSICA A

www.elsevier.com/locate/physa

Influence of partial slip on the peristaltic flow in a porous medium

T. Hayat, Q. Hussain, N. Ali*

Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan

Received 14 May 2007
Available online 21 February 2008

Abstract

In this paper, the slip effects are discussed on the peristaltic flow of a viscous fluid in a porous medium. A long wavelength approximation is used in the flow modelling. The solutions for stream function and axial velocity are constructed by employing the Adomian decomposition method. Numerical integration has been used for the pumping and trapping phenomena. Graphs illustrate the physical behavior. It is noted that the size of the trapped bolus decreases and its symmetry disappears for large values of the slip parameter. Further, the peristaltic pumping rate decreases by increasing the slip parameter.

Keywords: Peristaltic motion; Viscous fluid; Slip condition; Porous medium

1. Introduction

The importance of peristaltic flow phenomena had become more and more evident during the past few decades. This is perhaps due to several industrial and physiological applications of such flows. The first attempt regarding the peristaltic flow has been made by Latham [1]. Later, various workers have studied the peristaltic flow and now several solutions are available in the literature for such flows with considerations of the nature of the fluid, geometry of the tube, propagating waves, endoscope and inclusion of other physical effects such as magnetic fields, and porous media [2–15].

No attempt has been made yet to discuss the slip effects on the peristaltic flow in an asymmetric channel. Therefore the main purpose of the present paper is to present a theoretical analysis of slip effects on the peristaltic flow of a viscous fluid in a porous medium. The inadequacy of the no-slip condition is quite evident in polymer melts which often exhibit microscopic wall slip. The slip condition plays an important role in shear skin, spurt and hysteresis effects. The fluids that exhibit boundary slip have important technological applications such as in polishing valves of artificial heart and internal cavities. Here, the analysis has been carried out by considering slip effects on the asymmetric channel walls. The resulting problems for long wavelength approximation [2,3,5–14] are solved using the Adomian decomposition method (ADM) [16,17]. This method has already been used for the solutions of several other problems [18–24]. The paper is organized as follows. Section 2 includes the explanation of ADM briefly. The problem under consideration is formulated in Section 3. The solution to the problem is given in Section 4. Section 5

* Corresponding author. Tel.: +92 51 90642172.
E-mail address: nasirali_qau@yahoo.com (N. Ali).

0378-4371/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2008.02.040

includes previous results. The pumping and trapping phenomena are studied in Section 6. Section 7 consists of final remarks.

2. The Adomian decomposition method

To explain this method, we consider the following differential equation

$$Lu + Ru + Nu = g(x), \tag{1}$$

with prescribed conditions. In the above equation $u(x)$ is the unknown scalar function, L is the highest-order derivative which is assumed to be easily invertible, R is a linear differential operator of order less than L, Nu represents the nonlinear terms, and g is an inhomogeneous term. Applying the inverse operator L^{-1} to both sides of Eq. (1), and using the given conditions we have

$$u(x) = f(x) - L^{-1}(Ru) - L^{-1}(Nu), \tag{2}$$

in which the function $f(x)$ represents the terms arising from the integration of $g(x)$ and then using the prescribed initial or boundary conditions. For example, if $L = \frac{\mathrm{d}^3}{\mathrm{d}x^3}$, then L^{-1} is a three-fold integration, and $f(x) = u(0) + xu'(0) + \frac{x^2}{2!}u''(0) + L^{-1}g$.

The linear term $u(x)$ in terms of an infinite sum of components u_m is decomposed through the following equation

$$u(x) = \sum_{m=0}^{\infty} u_m(x). \tag{3}$$

The nonlinear operator Nu can be decomposed into an infinite series of polynomials as

$$Nu = \sum_{m=0}^{\infty} A_m(x). \tag{4}$$

The components of $u(x)$ can be determined recursively. $A_m(x)$ are the Adomian polynomials of $u_0, u_1, u_2, \ldots, u_m$ and satisfy

$$A_m = \frac{1}{m!}\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m}\left[N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0}, \quad m = 0, 1, 2, \ldots. \tag{5}$$

From Eqs. (2)–(4) we have

$$\sum_{m=0}^{\infty} u_m(x) = f(x) - L^{-1}\left(R\sum_{m=0}^{\infty} u_m\right) - L^{-1}\left(\sum_{m=0}^{\infty} A_m\right). \tag{6}$$

The above equation easily gives

$$\begin{aligned} &u_0 = f(x),\\ &u_{m+1} = -L^{-1}(Ru_m) - L^{-1}(A_m), \quad m \geq 0. \end{aligned} \tag{7}$$

All components are determinable since A_0 depends only on u_0, A_1 depends on u_0, u_1, etc. Moreover, since the series is commonly rapidly convergent, the m-term partial sum $\phi_m = \sum_{i=0}^{m-1} u_i$ could be the practical solution.

Regarding the convergence of the decomposition method and the detailed description of the Adomian decomposition and the modified decomposition algorithms, we refer the readers to the studies [25–27].

3. Problem formulation

Consider the two-dimensional channel of width $d_1 + d_2$ (Fig. 1) filled with an incompressible viscous fluid. The space between the channel walls is porous. The asymmetry in the channel flow is induced due to different amplitudes