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A hyperchaos generated from Lorenz system

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Abstract

This paper presents a four-dimension hyperchaotic Lorenz system, obtained by adding a nonlinear controller to Lorenz chaotic system. The hyperchaotic Lorenz system is studied by bifurcation diagram, Lyapunov exponents spectrum and phase diagram. Numerical simulations show that the new system's behavior can be convergent, divergent, periodic, chaotic and hyperchaotic when the parameter varies.

Keywords: Lorenz system; Hyperchaos; Lyapunov exponents; Bifurcation

1. Introduction

In 1979, Rössler proposed the conception of hyperchaos and presented hyperchaotic Rössler system [1]. Since it generates multiple positive Lyapunov exponents, hyperchaotic system's behavior is harder to predict than the general chaotic system. Thus hyperchaotic system is more valuable in secure communication and received a great deal of attention [2]. In recent years much research has been done in this area theoretically and experimentally [3,4]. Chen et al. [5] presented hyperchaotic Chen system based on Chen chaotic system; Lü et al. [6] proposed hyperchaotic Lü system based on Lü chaotic system; Nikolov et al. [7] presented modified hyperchaotic Rössler system; Gao et al. [8] gave another method to generate hyperchaos from Chen chaotic system. On synchronization, tracking and secure communication of hyperchaotic systems, a lot of research have been done and many methods have been presented [9–16]. In this paper, hyperchaos is generated from Lorenz chaotic system via adding a nonlinear controller to it. The new four-dimension system's behavior is studied by bifurcation diagram, Lyapunov exponents spectrum and phase diagram when the parameter varies.

2. Design of hyperchaotic Lorenz system

Lorenz system [17] is described as

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = cx - y - xz \\ \dot{z} = xy - bz, \end{cases} \tag{1}$$

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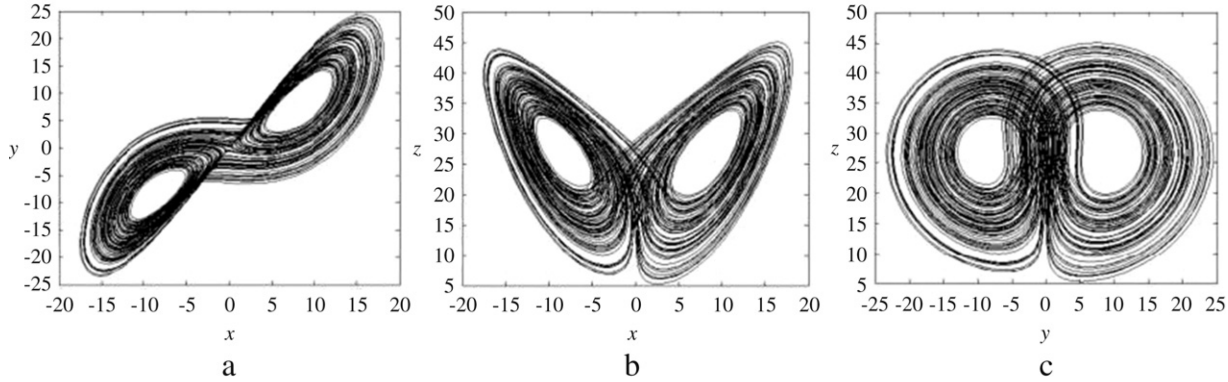

Fig. 1. The projections of Lorenz attractor.

when $a = 10$, $b = 8/3$, $c = 28$, Lorenz system exhibits a chaotic behavior, the projections of the chaotic attractor are shown in Fig. 1.

Add a nonlinear controller w to the first equation of system (1), let $\dot{w} = -yz + rw$, then we obtain a new system

$$\begin{cases} \dot{x} = a(y - x) + w \\ \dot{y} = cx - y - xz \\ \dot{z} = xy - bz \\ \dot{w} = -yz + rw. \end{cases} \tag{2}$$

In order to obtain hyperchaos, the important requirements are as follows:

(1) The system has dissipative structure.
(2) The minimal dimension of the phase space of an autonomous system is at least four.
(3) The number of terms in the equations giving rise to instability is at least two, of which at least one has a nonlinear function.

In system (2), r is the control parameter. Let $a = 10$, $b = 8/3$, $c = 28$, when

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -10 - 1 - 8/3 + r = r - 13.667 < 0,$$

system (2) can have dissipative structure, so theoretically the largest value of r is 13.667. Choose r near zero so that the previous dissipative structure can be maintained most, according to the method presented by Ramasubramanian et al. [18], we obtain when $r = -1$, the Lyapunov exponents: $\lambda_1 = 0.3381$, $\lambda_2 = 0.1586$, $\lambda_3 = 0$, $\lambda_4 = -15.1752$. It is obvious that system (2) exhibits a hyperchaotic behavior. The projections of the attractor are shown in Fig. 2.

3. Analysis of hyperchaotic Lorenz system

When $r > 0$, if $|r|$ becomes larger, the drive force in system (2) will be larger, system (2) tends to be not steady. On the contrary, when $r < 0$, if $|r|$ becomes larger, the dissipative force in system (2) will be larger, system (2) tends to be steady. In the numerical simulations, choose r near zero, we find that when $r > 0.17$, system (2) will diverge quickly and when r is negative, system (2) will converge to one of its equilibrium points if $|r|$ is relatively large. The simulation results are shown in Fig. 3.

From Fig. 3, we can see that sometimes system (2) needs long time to converge to one point. Through simulations of system (2)'s long-term behavior, we obtain that when $r < -6.43$, system (2) will converge to one of its equilibrium points.

In order to study the relation between r and system (2)'s behavior, we make the bifurcation diagram of system (2) for $-6.43 \le r \le 0.17$ in Fig. 4. $X_{\max}$ stands for the largest x in every unsteady period or steady period. When system (2)'s behavior is periodic, $X_{\max}$ has only one value or numbered values with certain r; When system (2)'s behavior is chaotic, $X_{\max}$ will have numberless values with certain r. According to the method presented by Ramasubramanian