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The Cox, Ross and Rubinstein tree model which includes counterparty credit risk and funding costs

Chadd B. Hunzinger[a,b,*,1], Coenraad C.A. Labuschagne[c,2]

[a] Rand Merchant Bank, 1 Merchant Place, Cnr Fredman Drive & Rivonia Road, Sandton 2196, South Africa
[b] Research Associate, Faculty of Economics and Financial Sciences, Department of Finance and Investment Management, University of Johannesburg, PO Box 524, Aucklandpark, South Africa
[c] Department of Finance and Investment Management, University of Johannesburg, Aucklandpark 2006, South Africa

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ABSTRACT

The binomial asset pricing model of Cox, Ross and Rubinstein (CRR) is extensively used for the valuation of options. The CRR model is a discrete analog of the Black–Scholes–Merton (BSM) model. The 2008 credit crisis exposed the shortcomings of the oversimplified assumptions of the BSM model. Burgard and Kjaer extended the BSM model to include adjustments such as a credit value adjustment (CVA), a debit value adjustment (DVA) and a funding value adjustment (FVA). The aim of this paper is to extend the CRR model to include CVA, DVA and FVA and to prove that this extended CRR model coincides with the model that results from discretising the Burgard and Kjaer model. Our results are numerically implemented and we also show that as the number of time-steps increase in the derived tree structure model, the model converges to the model developed by Burgard and Kjaer.

* Corresponding author. Tel.: +27 825648401.
E-mail address: chaddhunzinger@gmail.com (C.B. Hunzinger).
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1. Introduction

The 2008 credit crisis was a massive turning point for the world of finance and, in particular, for the pricing of financial derivatives. Before the crisis, pricing the value of a derivative was relatively straightforward. Universally, practitioners and academics agreed on the pricing method used to price a derivative. The method was simple, discount future expected cash flows under the risk-neutral measure to the present date using the risk-free rate. This method was derived from the fundamental theory laid down by Black, Scholes and Merton in the 1970s.

In practice, the binomial asset pricing model of Cox, Ross and Rubinstein (CRR) is extensively used for the valuation of options. The CRR model is a discrete-time analog of the continuous-time model of Black, Scholes and Merton (BSM) and is easy to implement in practice. The CRR and BSM models are covered in detail in the seminal papers of Black and Scholes (1973), Cox, Ross, and Rubinstein (1979) and Merton (1973).

Post the 2008 credit crisis, the pricing of financial derivatives has become more complex because many more factors need to be considered. These factors include the inability to borrow and invest cash at the risk-free interest rate and counterparty credit risk.

The risk-free rate is a theoretical rate of return of an investment with zero risk. It is the rate used to discount future expected risk-free cash flows to the present time. A crucial, but complex question is: What is the risk-free rate in the market? Possible proxies for the risk-free rate include the Treasury rate, the London Interbank Offered Rate (LIBOR) rate and the overnight indexed swap (OIS) rate. An OIS is an interest rate swap for which the overnight rate is exchanged for a fixed interest rate for a certain tenor.

According to Hull and White (2013a), the preferred choice of the risk-free rate by academics appears to be the Treasury rate. Prior to the 2008 credit crisis, practitioners assumed that a risk-free zero curve can be calculated from LIBOR rates, Eurodollar futures and swap rates. Post the 2008 credit crisis, banks seem to favour OIS rates for discounting collateralized transactions and LIBOR rates for discounting non-collateralized transactions. Hull and White (2013a) comprehensively discuss the LIBOR vs. OIS dilemma.

Defaults in countries such as Russia in 1998 and the more recent austerity measures taken by countries such as Greece and Spain to avoid default, indicate that Treasury rates are not risk-free. Furthermore, the difference between the interest rates on interbank loans and on short-term U.S. government debt (the TED spread) reached 450 basis points and the LIBOR-OIS spread 364 basis points in October 2008 (see Gregory, 2012, p. 286). The LIBOR-OIS spread has remained significant ever since. Prior to the 2008 credit crisis, this spread was stable and not significant. These shifts made it apparent that LIBOR incorporates an adjustment for the credit risk of the banks; therefore, LIBOR is an imperfect proxy for the risk-free rate. The OIS rate appears to be the best proxy for the risk-free rate.

Post the 2008 credit crisis, credit spreads of companies and, in particular, those of banks have increased dramatically. This heightened concerns about counterparty credit risks. Banks were no longer able to borrow at an unadjusted risk-free rate; in fact, there is no consensus on what the risk-free rate in the market should be.

Corporate clients are not risk-free, therefore banks charge their clients a credit value adjustment (CVA) in an over-the-counter (OTC) derivative trade. CVA is the price of the hedge used to mitigate the credit risk spread that is related to the corporate credit rating and the risk inherent in the transaction. More specifically, CVA is defined as the fair market value of the expected loss of an OTC derivative given that the opposite counterparty defaults. CVA is covered in detail by Jarrow and Turnbull (1995), Sorensen and Bollier (1994) and Duffie and Huang (1996).

Prior to the 2008 credit crisis, many banks' clients believed that banks were risk-free. Corporates never considered pricing CVA on the counterparty risk related to the bank into their transactions with banks. Post the 2008 credit crisis, banks are no longer seen as risk-free counterparties to corporate transactions. This resulted in the inclusion of a debit value adjustment (DVA) in the price of a derivative. DVA is the risk spread included in the derivative pricing that mitigate own credit risk in the transaction. More precisely, DVA is defined as the fair market value of the expected gain of an OTC derivative given own default. The origins of DVA are found in Duffie and Huang (1996). Gregory (2009) and Brigo and Capponi (2008) examine bilateral credit risk in general and derive DVA formally. In essence, DVA is an