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The deterministic shift extension and the affine dynamic Nelson–Siegel model

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ABSTRACT

The affine dynamic *Nelson–Siegel* model links the affine class of models with the *Nelson–Siegel* interpolation scheme of the yield curve. Its parameters are interpreted as the latent factors of the spot rate process driven by an affine diffusion. Using an appropriate specification of this diffusion, the yields become in form of the *Nelson–Siegel* model but an adjustment term is introduced. In this paper, the model is extended using a deterministic shift extension so as to perfectly fit the term structure and reduce the correction term. This enhancement allows to simulate the yield curve and the spot rate process consistently with the market data used for the calibration of the model. A numerical example discusses the calibration results of the original model and the proposed extension.

1. Introduction

The analysis of the yield curve allows to price financial contracts using two methodologies, as a consequence has led to numerous economic and econometric studies (see, for example Moura and Gaião (2014)). The discount factor is retrieved from this curve, hence the price of some instruments like bonds follows as a combination of discount factors. Since a market yield curve usually provides data

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associated to standard maturities, an interpolation method can be necessary for intermediate maturities. The *Nelson–Siegel* (NS, see Nelson and Siegel (1987)) and its extension, the *Nelson–Siegel–Svensson* (NSS, see Svensson (1994)), models are two schemes based on a parametrization of the yield to maturity. The empirical results show that these models produce satisfying results in terms of fitting quality of the market yield curves. When the models are estimated on a regular basis, then a time series of the parameters is constructed and statistically processed so as to forecast their future values. Such analysis leads to the dynamic *Nelson–Siegel* (DNS, see Diebold and Li (2006)) model. This model and its extensions can integrate data like macro-economic indicators in order to enhance their prediction capacity. However, the discount factor is not sufficient to price interest rate derivatives like options on bonds since such contracts need the distribution of the price at exercise date of their underlying. If the financial uncertainties are modelled with non-arbitrage stochastic models, then the market data reflects the expectations on the future state of the markets under the risk-neutral measure. It follows that the calibration of the models allows one to estimate the distributions of the financial uncertainties and/or to simulate them. One popular class of these models is the affine models relying on the affine diffusion processes. Although these models have less performing fitting results, they are able to price a variety of contracts with closed-form or analytical expressions, explaining their popularity for research and practical applications.

An unification of the NS and the affine models is performed using the affine dynamic *Nelson–Siegel* (AFDNS, see Christensen, Diebold, and Rudebusch (2011)) model. The parameters of the NS model are interpreted as the latent factors of the spot rate process and their evolution is controlled by an affine diffusion. The specification of this stochastic diffusion allows to obtain yields like those produced by the NS model but a correction term is added. The presence of this term is coherent with the non-arbitrage hypothesis assumed by the affine models and the lack of this assumption implied by the NS parametrization. The AFDNS model can be calibrated to market yield curve and the obtained parameters can be used to simulate yield curves with a variety of shapes. As a consequence, the model is suitable for the generation of economic scenarios that are required to the banks and insurance companies so as to value their solvency risk. Such financial risk management and regulation approaches are analysed on both theoretical (see, for example, Gupta, Akuzawa, and Nishiyama (2013)) and using econometrics methods (see, for example, McAleer, Jimenez-Martin, and Perez-Amaral (2013)). In this paper, the AFDNS model is extended introducing a deterministic shift (see Brigo and Mercurio (1998)) in order to fit perfectly the market yield curve. The shift is applied to the spot rate process, hence the expressions of the yield as well as the simulation methods are similar to the original model. This approach has been already introduced for an univariate affine model so as to improve its fitting ability. Moreover, under specific conditions, the adjustment term implied by the NS component can be reduced. The construction of the model is similar to the deterministic shift extensions, hence the model is named affine dynamic *Nelson–Siegel*++ (AFDNS++). Consequently, the proposed model grants the perfect fit to market data and allows to control the yield adjustment term.

This paper is organized following the steps of the construction of the AFDNS++ model. In Section (2), the NS, NSS and DNS as well as the affine models are reviewed and lastly the AFDNS model is presented. In Section (3), the deterministic shift extension framework and the AFDNS++ model are introduced. In order to illustrate the potential applications of the latter model, its theoretical strengths and weaknesses are discussed. Moreover, its practical implementation highlights other problems that have to be described. For this latter purpose, a numerical example (Section (4)) illustrates the calibration of the AFDNS and AFDNS++ models using market data provided by *European Central Bank* (ECB) and Section (5) concludes.

2. From the Nelson–Siegel to the affine dynamic Nelson–Siegel model

This section presents the AFDNS model of Christensen et al. (2011) following its logical construction. As its name suggests, this model relies on the NS interpolation scheme and the dynamics of its parameters are controlled by an affine process. Firstly the NS model and its extensions given by the NSS and DNS models are discussed (Section (2.1)), then the affine processes and some