



Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The equivalence of isothermal and non-isothermal power law distributions with temperature duality

Q1 Yahui Zheng^{a,b,*}, Jiulin Du^b^a Department of Physics, School of Science, Qiqihar University, Qiqihar City 161006, China^b Department of Physics, School of Science, Tianjin University, Tianjin 300072, China

ARTICLE INFO

Article history:

Received 2 November 2014

Received in revised form 1 January 2015

Available online xxxx

Keywords:

Temperature duality

Polytrope

Equilibrium configuration

ABSTRACT

The temperature duality states that the physical temperature and Lagrange temperature both have physical sense in the nonextensive system. By use of this assumption, the isothermal power law distribution and the non-isothermal power law distribution are equivalent to each other when the detailed balance is satisfied. Also, the polytropic equation in stellar system and self-gravitating gaseous system can be deduced from both of these two distributions. This indicates that the polytropic system exhibits some 'equilibrium' configuration which, in the stellar system, is probably the result of so called 'violent relaxation'.

© 2015 Published by Elsevier B.V.

1. Introduction

The nonextensive statistical mechanics (NSM) firstly proposed by Tsallis in 1988 [1] has been proved to be an effective theory, supported by the observations of the solar wind and space plasmas [2–7]. Up to now, it has been extensively accepted and has been applied so many fields such as astrophysics, biology, and economics. However, one fundamental problem about how to define the temperature in NSM remains, which is related to another question that how to understand the essence of nonextensivity in different systems.

The assumption of temperature duality probably can resolve this problem to some extent, which states that, in the nonextensive system, the temperature concept should be split into two concepts: the physical temperature [8], associated with the nonextensivity of the system, and the Lagrange temperature as the inverse of Lagrange multiplier, related to the local collision effect of molecules. They are linked through the expression,

$$T_q = C_q T. \quad (1)$$

The quantity T_q is the physical temperature, and the other one T is the Lagrange temperature, which is directly related to the derivative of Tsallis entropy with respect to the total energy of the system [8]. The coefficient in (1) is called Tsallis factor, defined as

$$C_q \equiv \sum p_i^q = 1 + (1 - q)S_q/k, \quad (2)$$

where p_i is the ensemble distribution function of the system, and S_q is the Tsallis entropy of system, with such an expression [1],

$$S_q = k \frac{\sum p_i^q - 1}{1 - q}. \quad (3)$$

* Corresponding author at: Department of Physics, School of Science, Qiqihar University, Qiqihar City 161006, China.
E-mail address: zhengyahui1979@163.com (Y. Zheng).

In the ensemble theory, the physical temperature is used to determine the last state of evolution of an isolated nonextensive system, which is ordinarily a nonequilibrium stationary state with detailed balance. Any measurement to ‘temperature’ is carried out in the local collision equilibrium; therefore the Lagrange temperature related to local collision effect is real in physics. It is measurable experimentally. In this sense Lagrange temperature is equivalent to the classical temperature concept. Contrary to this, the physical temperature is immeasurable in physics. However, this does not mean the physical temperature is useless, because it is the state parameter of the last state of evolution of an isolated system. As a state parameter, it is homogeneous in the whole system. That is why we call the last state of evolution ‘Tsallis equilibrium state’.

This idea of temperature duality is firstly applied into the nonextensive gas [9], in view of which the second virial coefficient is explained as the power law form of the Lagrange temperature and the power index is proportional to the nonextensive parameter $(1 - q)$. Therefore, one finds a way to relate the nonextensive parameter to the short range potential of molecules. The use of this novel idea is far beyond this. In the next context of this paper, we would apply this idea into two different systems, to discuss the relation of two power law distributions: one is isothermal and another is non-isothermal.

2. The stochastic dynamic system

Now consider a stochastically dynamic system, where, for simplicity, the Brownian motion is set to be one dimensional. The Brownian particle, moving in a potential, yields to the two-variable Langevin equations for position and momentum (x, p) , that is,

$$\frac{dx}{dt} = \frac{p}{m}, \quad \frac{dp}{dt} = -\frac{dV(x)}{dx} - \gamma(x, p)p + \eta(x, p, t) \quad (4)$$

where m is the mass of the particle, the γ is the friction coefficient. The stochastic force is also assumed to be Gaussian. Therefore,

$$\langle \eta(x, p, t) \rangle = 0, \quad \langle \eta(x, p, t)\eta(x, p, t') \rangle = 2D(x, p)\delta(t - t'). \quad (5)$$

The quantity D is the correlation strength of stochastic force. We assume the Brownian particle moves in an inhomogeneous complex medium; therefore the friction coefficient and correlation strength are both inhomogeneous in the phase space (x, p) [10]. In order to interpret the inhomogeneity of phase space, we must assume that there is long range correlation between any two phase points. Because the short range correlation cannot keep the inhomogeneous property of the phase space. This seems to imply that the potential in (4) is associated with a long range external field force. It is important for the discussion later on.

Applying the Zwanzig rule [11] (also called the backward Ito rule), the Fokker–Planck equation about noise-averaged distribution, corresponding to Langevin equations (4) and (5), can be written as

$$\frac{\partial f}{\partial t} = -\frac{p}{m} \frac{\partial f}{\partial x} + \frac{\partial}{\partial p} \left(\frac{dV(x)}{dx} + \gamma(x, p)p \right) f + \frac{\partial}{\partial p} \left(D(x, p) \frac{\partial f}{\partial p} \right). \quad (6)$$

The quantity $f(x, p, t)$ is the distribution in phase space (x, p) at time t . We concern its stationary state form, i.e.,

$$-\frac{p}{m} \frac{\partial f_s}{\partial x} + \frac{\partial}{\partial p} \left(\frac{dV(x)}{dx} + \gamma(x, p)p \right) f_s + \frac{\partial}{\partial p} \left(D(x, p) \frac{\partial f_s}{\partial p} \right) = 0. \quad (7)$$

In order to our aim, we consider the special case of the above equation, when the condition detailed balance is approached. In this case, the above equation can be further divided into these two equations,

$$-\frac{p}{m} \frac{\partial f_s}{\partial x} + \frac{dV(x)}{dx} \frac{\partial f_s}{\partial p} = 0 \quad (8)$$

$$\frac{\partial}{\partial p} [\gamma(x, p)p f_s] + \frac{\partial}{\partial p} \left[D(x, p) \frac{\partial f_s}{\partial p} \right] = 0. \quad (9)$$

Eq. (9) shows the sum of drift term and diffusion term in the phase space tends to zero under the condition of detailed balance.

Along the same step as Ref. [10], we define the following function

$$g(x, p) = \frac{D(x, p)}{m\gamma(x, p)}. \quad (10)$$

Expand this function in the momentum space, based on the same discussion as Ref. [12], adopting the first two terms in the Taylor expansion about momentum, one has

$$g(x, p) = \beta^{-1}(x) - \lambda \frac{p^2}{2m} = \beta^{-1}(x) \left(1 - \lambda \beta \frac{p^2}{2m} \right). \quad (11)$$

Download English Version:

<https://daneshyari.com/en/article/975010>

Download Persian Version:

<https://daneshyari.com/article/975010>

[Daneshyari.com](https://daneshyari.com)