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Optimal control of a delayed SLBS computer virus model

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HIGHLIGHTS

- Propose a delayed SLBS computer virus model.
- Investigate existence of the optimal control strategy.
- Compute cost value when choosing different delay and control strategy.
- Give numerical simulations to show the effectiveness of the strategy.

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ABSTRACT

In this paper, a delayed SLBS computer virus model is firstly proposed. To the best of our knowledge, this is the first time to discuss the optimal control of the SLBS model. By using the optimal control strategy, we present an optimal strategy to minimize the total number of the breakingout computers and the cost associated with toxication or detoxication. We show that an optimal control solution exists for the control problem. Some examples are presented to show the efficiency of this optimal control.

1. Introduction

Computer virus aroused in the 1980s has become a great threat to our work and daily life. Especially, in recent years, with the development of hardware and software technology and the popularity of computer networks, this threat has become more serious. For example, nowadays computer virus is capable of acquiring personal data from network users, such as passwords and bank accounts, causing severe damages to individuals and corporations. So it is urgent to propose various mathematical models to analyze the spread and control of computer virus. Due to the high similarity between computer virus and biological virus, various computer virus propagation models were proposed [1–13]. Many classic epidemic models, such as the SIS model [1], SIR model [2,3], SIRS model [4–8,14,15], SIEIS model [9], SIC model [10] and SVEISV model [11], have been borrowed to depict the spread of a computer virus. However, as Yang and Yang [12] pointed out, many previous models lay emphasis on the similarity between computer virus and infectious disease and the majority of them more or less neglect the intrinsic difference between them. In their opinion, in the real world there exists no infected computer at all that has no infectivity. Equivalently, there exists no exposed computer, implying that a rational epidemic model of computer

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virus should have no E compartment. Moreover, an infected computer will be referred to as latent or breaking-out depending on whether all viruses staying in it are in their respective latent periods or at least one virus staying in it is in its breaking-out period. A model that aims to capture the spread of a large family of computer virus should not possess a permanent R compartment. Based on the previous discussion, the authors classify all the computers as three categories: uninfected computers (S), latent internal computers (L) and breakingout (B) computers. Let $S(t)$, $L(t)$ and $B(t)$ denote the numbers of S, L and B computers at time t, respectively. Based on some assumptions, in Ref. [12], the SLBS model is formulated as

$$\begin{cases} \dfrac{\mathrm{d}S(t)}{\mathrm{d}t} = \mu_1 + \gamma_1 B(t) + \gamma_2 L(t) - f(L(t)+B(t))S(t) - (\delta_1+\theta)S(t), \\ \dfrac{\mathrm{d}L(t)}{\mathrm{d}t} = \mu_2 + f(L(t)+B(t))S(t) + \theta S(t) + \gamma_3 B(t) - (\alpha+\gamma_2+\delta_1)L(t), \\ \dfrac{\mathrm{d}B(t)}{\mathrm{d}t} = \alpha L(t) - (\gamma_1+\gamma_3+\delta_1+\delta_2)B(t). \end{cases} \tag{1.1}$$

The detailed meaning of the coefficients in (1.1) in Ref. [12] is as follows. S computers and L computers are connected to the Internet at constant rate μ_1 and μ_2, respectively. Every B computer is cured with constant probability γ_1, every L computer is cured with constant probability γ_2, and every B computer is partially cured, that is, becomes an L computer, with constant probability γ_3. S and L computers are disconnected from the Internet with constant probability δ_1. Every B computer is disconnected from the Internet with constant probability $\delta_1+\delta_2$. Each S computer is infected with constant probability θ and every L computer becomes a B computer with constant probability α. Due to the contact with L or B computers, at time t every S computer becomes an L computer with probability $f(L(t)+B(t))$, where the function f is continuously differentiable.

Furthermore, a series of new epidemic models, known as the SLBS models, have been suggested [13,16–19]. For example, the authors in Ref. [13] derive the following computer virus propagation model

$$\begin{cases} \dfrac{\mathrm{d}S(t)}{\mathrm{d}t} = \delta + \gamma_1 L(t) + \gamma_2 B(t) - \beta(L(t)+B(t))S(t) - \delta S(t), \\ \dfrac{\mathrm{d}L(t)}{\mathrm{d}t} = \beta(L(t)+B(t))S(t) - (\alpha+\gamma_1+\delta)L(t), \\ \dfrac{\mathrm{d}B(t)}{\mathrm{d}t} = \alpha L(t) - (\gamma_2+\delta)B(t) \end{cases} \tag{1.2}$$

and the qualitative properties of this model are fully studied. In 2014, Yang et al. [16] get over some flaws of previous models and present a novel epidemic model as follows.

$$\begin{cases} \dfrac{\mathrm{d}S(t)}{\mathrm{d}t} = \mu_1 + \gamma_1 L(t) + \gamma_2 B(t) - (\beta_1 L(t)+\beta_2 B(t))S(t) - (\delta+\theta)S(t), \\ \dfrac{\mathrm{d}L(t)}{\mathrm{d}t} = \mu_2 + (\beta_1 L(t)+\beta_2 B(t))S(t) + \theta S(t) - (\alpha+\gamma_1+\delta)L(t), \\ \dfrac{\mathrm{d}B(t)}{\mathrm{d}t} = \alpha L(t) - (\gamma_2+\delta)B(t). \end{cases} \tag{1.3}$$

In Ref. [16], with the aid of the theory of asymptotically autonomous systems as well as the generalized Poincare–Bendixson theorem, the unique endemic equilibrium of the proposed model is shown to be globally asymptotically stable. In Ref. [17], in view of the scale-free property of the Internet, a novel epidemic model of computer virus is advised. The corresponding spreading threshold is determined. The virus-free equilibrium is proved to be globally asymptotically stable provided the threshold is below the unity, whereas the permanence of the virus equilibrium is shown if the threshold exceeds the unity. However, here we have to point out that the above mentioned literatures only emphasize on qualitative analysis such as seeking the so-called basic reproduction number and discussing the existence and the stability of equilibria and periodic orbits. Actually, another important way to control virus outbreak is the optimal control theory, which pay attention to define a strategy to control the disease and obtain the best possible result.

On the other hand, due to the time cost needed to develop new patches, there is a delay τ from the time an L or B computer is cured. Moreover, due to the intrinsic latent period of virus, there is delay τ from the time an S computer is infected to the time this computer become latent or break out. Motivated by the above, we propose the following controlled computer virus model with delay.

$$\begin{cases} \dfrac{\mathrm{d}S(t)}{\mathrm{d}t} = \mu_1 + \gamma_1 L_\tau(t) + \gamma_2 B_\tau(t) - (\beta_1 L(t)+\beta_2 B(t))S(t) - (\delta+\theta)S(t) + \omega u(t)B(t), \\ \dfrac{\mathrm{d}L(t)}{\mathrm{d}t} = \mu_2 + (\beta_1 L_\tau(t)+\beta_2 B_\tau(t))S_\tau(t) + \theta S(t) - (\alpha+\gamma_1+\delta)L(t) + (1-\omega)u(t)B(t), \\ \dfrac{\mathrm{d}B(t)}{\mathrm{d}t} = \alpha L_\tau(t) - (\gamma_2+\delta)B(t) - u(t)B(t) \end{cases} \tag{1.4}$$

where $x_\tau(t) = x(t-\tau)(x = S, L, B)$. The reason why the inclusion of time delay τ into susceptible, latent and breakout in transmission rate, only on the second equation is as follows. As we know, at time t, the density of L increases because S