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New multi-parametric analytical approximations of exponential distribution with power law tails for new cars sells and other applications

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HIGHLIGHTS

- We made 3- or 4-parametric PDF with exponential main part and Pareto asymptotic tail.
- We traced sells of new cars in Russia and in the USA and distributions over prices.
- We motivate the correspondence of the distributions of new car sells and incomes.
- The introduced PDF described above distributions better than standard fits.

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ABSTRACT

A multi-parametric family of exponential distributions with various power law tails is introduced and is shown to describe adequately the known distributions of incomes and wealth as well as the recently measured distributions of new car sales. The three or four-parametric families are characterized by effective temperature in the exponential part, the power exponent in the power-law asymptotic part, the coefficient for the transition between the above two parts, and the starting value, if it is not equal to zero. Since the new car sales distributions are found to correspond to known distributions of incomes, the latter may be inferred from the former.

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0. Introduction

Elementary physical processes such as molecular dissociation, atomic excitation and ionization, etc. are determined by random values of particle velocities that typically distanced considerably from the maximum velocity of the random value distribution density. Moreover, the random value distribution in its asymptotic part (or distribution "tail") may be different from the main (usually exponential) part. The problem of exponential distributions with asymptotic power-law tails is relevant to various physics problems such as that of describing the distribution of hot electrons in plasmas [1]. The same type of distribution behavior is known to arise in econophysics in the case of the distribution of income and wealth of citizens in countries with advanced tax systems [2]. In physics, the existence of distribution tails may be due by various effects including microfields in plasmas [3], the effective change of dimensions (fractality) [4], mixed elementary forces [5], or the power-law character of the basic distributions [6]. Q2

The exponential part of the distribution characterized by some normalization coefficient as well as the power exponent in the tail part is routinely calculated using various software such as Origin. Two difficulties are encountered in the process.

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The first one stems from the fact that the distribution near zero is not known or does not tend to 1 if normalized. The second difficulty lies in describing the transition between the exponential part and the power-law tail part of the distribution. The number of experimental points in the tail usually being fairly limited, setting up an approximation of the power law on the basis of only a few sample points may be a serious problem. It is demonstrated below that its solution leads to an error of around 0.5 in the power exponent as compared to the approximation suggested in the present paper.

The number of experimental points varies depending on the area of study. In income and wealth distributions, empirical data are provided by statistical agencies. The Internal Revenue Service (IRS) in the USA provided 18 points of income distribution (for 2004 in positive part) [7], Annual Abstract of Statistics in the UK provided 9 points (for 2003–2004) [8], Russian Federal State Statistical Service supplied no data on income distribution tails whatsoever (for 2011 and earlier, see Ref. [9] and below). The tail of the energy distributions are normally observed in physics, in the experiments on the production of hot electrons in laser plasmas, but, the numbers of experimental points being small, tail dependence is usually hard to determine. As a result, attempts were made to describe the hot electrons distribution theoretically (see, for example Refs. [10,11]).

In econophysics, the pertinent subject of study does not have to be limited to income and wealth distributions, and can be extended to include the distributions of sales of certain types of products. Indeed, the price ranges in the cases of a number of types of products can be extremely wide (up to several orders of magnitude), and the distribution of sales of such products can be expected, similarly to those discussed above, to have tail parts. New cars are an appropriate example of a type of product selling within a wide price bracket.

The sales of new cars can be explored in the above framework considering that their prices in Russia in 2012 started from RUR 200 000 (RUR is Russian Ruble) with virtually no upper bound. The number of cars sold at $100 000 (about RUR 3,000,000) and above was over 100,000, the picture being similar in other countries [12,13].

At the same time, it was found [13] that the range of new car prices covers the range of personal incomes. Therefore, the distribution of prices of new cars sales can be expected to reflect the distribution of personal incomes in a given country. To prove the hypothesis, the official income statistics from countries with well-established economies is compared against the amounts of money spent to acquire new cars in the same countries. This approach may be used to access (at least in part) on the quantitative level the distribution of personal of incomes, as the percentage of personal incomes spent to buy new cars appears to be the same across the developed economies.

We start from the introduction of the new family of multi-parametric functions describing distributions which have an "exponential body" and power-law tail. As the next step, we apply the results obtained to describe the income and brand-new car sales distribution in a developed country (in the USA), and explore the brand-new car sales distributions in modern Russia. Findings from the latter exploration are used below to estimate the actual personal incomes in Russia.

1. Multi-parametric family of curves with exponential main part and power law tail

To define the general form of the desired distribution, one may proceed from the results presented in Ref. [14] as a starting point. According to Ref. [14], that the sum of a large quantity N of random values similarly distributed with the probability density function (PDF) of the Student's (generally, non-integer) type $\sim z_0^{2\beta}/(z_0^2+f^2)^{2\beta}$ has the distribution of the Gaussian form for comparably small values of fluctuations f:

$$W_G(f) \approx \frac{1}{\sqrt{\pi}}\exp(-f^2)$$

and $\sim 1/f^{-2\beta}$ for large f (z_0 being a normalization constant, the sum is treated as random walks in Ref. [14]). The obvious mathematical generalization to get the exponential part with power-law tail is to perform the transformation $f^2 \to R/T$ (here T can be interpreted as an effective "temperature"). Upon switching from parameters N, z_0, β to parameters θ, T, β, the transformation yields the curve with the exponential main part and a transition to power law at the tail in an explicit form of a PDF:

$$W_{T\beta\theta}(R) = \frac{1}{\sqrt{\pi T}}\int_0^\infty \cos(x\sqrt{R})\left\{\frac{2}{\Gamma(\beta-1/2)}\left[(\beta-3/2)\frac{xT}{4\theta}\right]^{\beta/2-1/4} \mathrm{K}_{\beta-1/2}\left[\sqrt{(\beta-3/2)\frac{xT}{4\theta}}\right]\right\}^{\theta} \mathrm{d}x. \quad (1)$$

Here R is variable, Γ is the gamma-function, $\mathrm{K}_{\beta-1/2}$ is the modified Bessel function of the 2nd kind (also known as "McDonald function"). Note that the above integral converges for any $\beta > 3/2$. It does not exist for smaller β. The fact that $W_{T\beta\theta}$ is positive for any $\beta > 3/2$ is followed from the deduction of an initial PDF for R in Ref. [14].

It is clear that the expression for PDF like (1) can involve dependences more complicated than the square root function. For the power-law function $F(R) \sim R^{\xi}$, ($\xi \neq 1/2, 1$), the final distribution (similar to that expressed by Eq. (1)) will have the stretch exponent main part with a power-law tail. Even more artificial forms of (1) may prove useful (see also Ref. [15] for more on functional random walks).

The approximation of Eq. (1) for comparably small R (up to several values of T) is easily reduced to a dependence on parameter T only

$$W_T(R) \cong \frac{1}{T}\exp\left(-\frac{R}{T}\right). \quad (2)$$

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