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The returns and risks of investment portfolio in stock market crashes[☆]

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HIGHLIGHTS

- The returns and risks of investment portfolio in stock market crashes were investigated.
- Both the maximum dispersion of investment portfolio and an optimal stop-loss position maximally enhance the stability of returns.
- A worst dispersion is associated with the maximum risks.
- The increasing stop-loss position enhances the risks.

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ABSTRACT

The returns and risks of investment portfolio in stock market crashes are investigated by considering a theoretical model, based on a modified Heston model with a cubic nonlinearity, proposed by Spagnolo and Valenti. Through numerically simulating probability density function of returns and the mean escape time of the model, the results indicate that: (i) the maximum stability of returns is associated with the maximum dispersion of investment portfolio and an optimal stop-loss position; (ii) the maximum risks are related with a worst dispersion of investment portfolio and the risks of investment portfolio are enhanced by increasing stop-loss position. In addition, the good agreements between the theoretical result and real market data are found in the behaviors of the probability density function and the mean escape time.

1. Introduction

The applications of statistical physics to investigate the financial markets have obtained more and more attentions and given rise to a new field called "econophysics" in recent years [1,2]. The original model is the geometric Brownian motion model [3,4], but it cannot reproduce some statistical characteristics from actual financial data, such as, the fat tails [5,6], long range memory and clustering of volatility [7]. Therefore, to make up these deficiencies, such as the ARCH model [8], GARCH model [9], Heston model [10] and many valuable models are proposed. In particular, statistical characteristics of stock prices obtained from actual financial data are well described by the Heston model. For example, see the probability distribution of returns for the three major stock-market indexes (Nasdaq, S&P500, and Dow-Jones) [11], the exponential distribution of financial returns obtained from actual financial data [12], the probability density distribution of the logarithmic returns of the empirical high-frequency data of DAX and its stocks [13] or the typical price fluctuations of the Brazilian São Paulo Stock

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Exchange Index [14]. Then, since a Langevin equation approach to a model for stock market fluctuations and crashes was analyzed [15], the stock market crashes have been widely discussed with Heston model, and the noise enhanced stability phenomenon has been widely discovered. For example, see the mean escape time in financial markets [16,17], the statistical properties of the hitting times in different models for stock market evolution [18,19], statistical properties of escape times for stock price returns in the Wall Street market [20], exact expressions for the survival probability and the mean exit time [21,22], the effects of the delay time [23,24].

Moreover, the studies on an appropriate investment portfolio in the actual trade in financial market have been widely discussed, such as a stochastic optimal control approach of geometric Brownian motion model on the investment optimization [25], the optimal investment in an incomplete semimartingale market [26,27], a goal programming approach to combine various forecasts by using investment portfolio return [28], the optimal investment portfolio in renewable energy [29] and the returns and risks of investment portfolio [30]. Meanwhile, the daily international portfolio flows into and out of 44 countries from 1994 through 1998 [31], a portfolio of twenty-two major banks in Asia and the Pacific [32], a long VIX investment [33] and so on in financial crisis were also done. However, the above studies of investment portfolio are deficient in the dynamic model of stock prices. Hence, the roles of investment portfolio in stock market crashes need to be further discussed.

In this paper, we use the modified Heston model with an effective cubic potential, proposed by Spagnolo and Valenti [16–19], to describe the investment portfolio and stock market crashes, respectively. Then we build a dynamical model of investment portfolio constructed by two stock in stock market crashes. The probability density function (PDF) of returns is employed to analyze the returns of portfolio [24,30]. The risks of portfolio are pictured by the mean escape time of the dynamical model representing the time of the stock price staying in a price range in physical sense [16–18,21]. In addition, the remainder of this paper is organized as follows. In Section 2, a model of the investment portfolio with stock market crashes is presented. In Section 3, we introduce the returns of portfolio in financial crisis. The risks of portfolio are given via the analyses of the mean escape time in Section 4. In Section 5, the comparison between the theoretical result and real market data is given. In Section 6, a brief conclusion ends the paper.

2. The model of investment portfolio with stock market crashes

In order to conveniently investigate the roles of investment portfolio in the case of stock market crash, we use an effective potential with an unstable state and two different dynamical regimes to describe stock market in normal activity and extreme days [16–19] and discuss the returns and risks of investment portfolio of two uncorrelated stocks [30]. Then, the dynamic model of rate of investment portfolio $C(t)$ becomes:

$$\begin{aligned}
&C(t) = r_1 \exp(x_1(t) - x_1(0)) + (1 - r_1)\exp(x_2(t) - x_2(0)),\\
&\mathrm{d}x_1(t) = -\left(\frac{\partial U(x_1)}{\partial x_1} + \frac{\nu_1(t)}{2}\right)\mathrm{d}t + \sqrt{\nu_1(t)}\mathrm{d}W_1(t),\\
&\mathrm{d}\nu_1(t) = a_1(b_1 - \nu_1(t))\mathrm{d}t + c_1\sqrt{\nu_1(t)}\mathrm{d}Z_1(t),\\
&\mathrm{d}x_2(t) = -\left(\frac{\partial U(x_2)}{\partial x_2} + \frac{\nu_2(t)}{2}\right)\mathrm{d}t + \sqrt{\nu_2(t)}\mathrm{d}W_2(t),\\
&\mathrm{d}_2\nu(t) = a_2(b_2 - \nu_2(t))\mathrm{d}t + c_2\sqrt{\nu_2(t)}\mathrm{d}Z_2(t),
\end{aligned} \tag{1}$$

where r_1 is the initial rate of the first stock on the total investment portfolio, $x_i(t)$ describes the log of the ith stock price ($i = 1, 2$), $\nu_i(t)$ denotes the volatility of the stock price, a_i denotes the mean reversion of the $\nu_i(t)$, b_i denotes the long-run variance of the $\nu_i(t)$, c_i is often called the *volatility of volatility* i.e., it is the amplitude of volatility fluctuations. $x_1(0) = -1.25$, $x_2(0) = -1.25$, the effective cubic potential U is $U(x) = px^3 + qx^2$ with $p = 2$ and $q = 3$ (see Fig. 1). For the convenience of analysis, $\mathrm{d}W_i(t)$ and $\mathrm{d}Z_i(t)$ are considered as uncorrelated Wiener processes and have the following statistical properties:

$$\begin{aligned}
&\langle \mathrm{d}W_i(t)\rangle = \langle \mathrm{d}Z_i(t)\rangle = 0,\\
&\langle \mathrm{d}W_i(t)\mathrm{d}W_j(t')\rangle = \langle \mathrm{d}Z_i(t)\mathrm{d}Z_j(t')\rangle = \delta_{i,j}\delta(t-t')\mathrm{d}t,\\
&\langle \mathrm{d}W_i(t)\mathrm{d}Z_i(t')\rangle = \langle \mathrm{d}Z_i(t')\mathrm{d}W_i(t')\rangle = 0.
\end{aligned} \tag{2}$$

For the effective cubic potential $U(x)$, we can directly see from Fig. 1 that the $U(x)$ has one stable state at $x_s = 0$, and an unstable state at $x_u = -1.0$, and if x_l is equal to -1.5, the $U(x_l)$ is equal to $U(x_s)$. In terms of probability, with the starting position $x_0 \in [x_l, x_u]$, a Brownian particle tends to enter into the right area rather than escape from as long as the particle has entered. This behavior can be useful to describe the stock price crashes from a given range of prices in extreme days. Thus the time of stock market bubble or crash, such as the analysis of Bonanno et al. [16], is investigated via the mean escape time that the price starts from the initial position $C(0) = 100\%$ [i.e., $x_1(0) = -1.25$ and $x_2(0) = -1.25$ in the unstable area ($[x_l, x_u]$)] to absorbing barrier C_a. If $C_a = 10\%$, the investment portfolio with $C \le 10\%$ is liquidated due to under margin. The absorbing barrier C_a therefore pictures the stop-loss price of investors. Particular discussion on escape time and different dynamical regimes see Refs. [34,35].