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Consensus of heterogeneous multi-agent systems with switching jointly-connected interconnection[☆]

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HIGHLIGHTS

- We give a new method to investigate the consensus problem.
- We give a new model transformation.
- We give a new consensus condition for heterogeneous multi-agent systems.
- Consensus can be achieved when the feedback gain of velocity is large enough.
- The realization of consensus is unrelated to the feedback gain of position.

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ABSTRACT

In this paper, the consensus problem of heterogeneous multi-agent systems with switching jointly-connected interconnection and a leader is considered. Firstly, by a model transformation, the original closed-loop system is turned into an equivalent system. And then, by applying the matrix theory and Lyapunov directed method, the convergence of the multi-agent systems is analyzed, a sufficient condition for consensus of systems is derived when the communication topologies are jointly-connected. Finally, simulation results are provided to demonstrate the effectiveness of presented results.

1. Introduction

In the past decade, the consensus problem of multi-agents systems has attracted more and more attention, partly due to its important applications in practical systems, such as unmanned vehicles and automated highway systems, etc. For multi-agent systems with first-order, second-order and high-order integrator, many rules have been designed based on the information of each agent and its neighbors [1–22]. For example, Vicsek et al. proposed a simple model for phase transition of a group of self-driven particles and numerically demonstrated complex dynamics of the model [3], and then Jadbabaie et al. provided a theoretical explanation for the consensus behavior of the Vicsek model using graph theory [4]. And then, the robust H_∞ consensus problems for first-order, second-order and high-order multi-agent systems with external disturbances were studied in Refs. [15–18]. And some conditions are derived to make all agents reach consensus with H_∞ performance.

Most of results of consensus problem are on multi-agent systems with the same-order dynamics. Recently, the consensus of heterogeneous multi-agent systems has been received more and more attention because of the dynamics of the agents

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coupled with each other are different in the practical systems. The consensus problem of heterogeneous multi-agent systems with linear consensus protocol and saturated consensus protocol was studied in Ref. [23], some sufficient conditions for consensus are established when the communication topologies are undirected connected graphs. Then the consensus problem of heterogeneous multi-agent systems with directed communication graphs was investigated in Ref. [24]. And the consensus problem of heterogeneous multi-agent systems composed of first-order, second-order and nonlinear Euler–Lagrange agents was considered in Ref. [25]. Most of literatures on the heterogeneous multi-agent systems assume that the topologies are connected. In Ref. [26], the consensus of heterogeneous multi-agent systems with switching directed topologies and a virtual leader is studied. They found that the system achieves the consensus if the union of the directed graphs has a directed spanning tree. In practical systems, however, interaction topologies between individual agents may change over time. And the time varying topologies maybe not connected at any moment. In the study of variable topologies, joint connection is an important condition. For homogeneous multi-agent systems, Hong et al. studied the consensus problem of multi-agent systems with switching jointly connected topologies [19]. Lin and Jia proved the convergence of a class of second-order leaderless multi-agent systems with jointly connected topologies [20]. However, to the best of our knowledge, there is little work to deal with the consensus problem of heterogeneous multi-agent systems. This problem is difficult and the existing approaches, e.g. Refs. [19,20], cannot be applied straightforward here.

In this paper, we extend the consensus result of homogeneous multi-agent systems to heterogeneous multi-agent systems, which is composed of first-order and second-order integrator agents. Different from the literatures on the consensus problem of homogeneous multi-agent systems, we assume that the topologies is jointly connected herein. A new method is given to deal with the consensus problem of heterogeneous multi-agent systems. First, neighbor-based protocols are adopted to make the heterogeneous multi-agent systems realize consensus. Second, the original closed-loop system is changed into an equivalent system by a coordinate transition. Then, by applying matrix theory and Lyapunov method, some sufficient conditions for consensus are deduced, under which the positions of all agents converge to the position of the leader and the velocities of second order agents converge to zero. Finally, simulation results are provided to verify the effectiveness of the previous results.

2. Preliminaries

2.1. Graph theory

To solve the coordination problems, graph theory is useful (see Ref. [27] for details). Consider a dynamical system consisting of n agents. With regarding the n agents as the vertices $V = \{v_i, i = 1, 2, \dots, n\}$, the interconnection topology of n agents can be conveniently described by an undirected graph $G = \{V, \varepsilon\}$, where $\varepsilon \subset V \times V$ is the set of edges of the graph. (v_i, v_j) defines one of the graph's edges if v_i and v_j are neighbors. $N_j(t) = \{ i \mid (v_i, v_j) \in \varepsilon\}$ denotes the set of labels of those agents which are neighbors of agent i $(i = 1, 2, \dots, n)$ at time t. Let $A = [a_{ij}] \in R^{n\times n}$ is the weighted adjacency matrix of the graph G, and $D = diag\{d_1, d_2, \dots, d_n\} \in R^{n\times n}$ is its degree matrix. Then the Laplacian of the weighted graph is defined as $L = D - A$, which is symmetric. Let v_0 represents the leader, and the connection between the agents and the leader is directed, we namely, there are only edges from some agents to the leader, but there is no edge from the leader to any agent. Then we get a graph $\overline{G}$ with vertex set $\overline{V} = V \sqcup \{v_0\}$. About 'the graph $\overline{G}$ is connected', we mean that there is at least one agent in each connected component of G connected to the leader.

A union graph of a collection of simple graphs $\overline{G}_1, \overline{G}_2, \dots, \overline{G}_N$, $(N \geq 1)$, with the same vertex $\overline{V}$, is a simple graph, denoted by $\overline{G}_{1,\dots,N}$ with vertex set $\overline{V}$ and edge set being the union of edge sets of all the graphs in the collections. And, the collection, $\overline{G}_1, \overline{G}_2, \dots, \overline{G}_N$, is jointly connected if and only if its union graph $\overline{G}_{1,\dots,N}$ is connected.

2.2. System model

Consider a heterogeneous multi-agent system composed of first-order and second-order integrator agents. The number of agents is n. Assume that there are m second-order integrator agents $(m < n)$. And each second-order agent dynamics is given as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t), \quad i \in \mathcal{I}_m, \end{cases} \tag{1}$$

where $x_i \in \mathcal{R}$, $v_i \in \mathcal{R}$ and $u_i \in \mathcal{R}$ are the position, velocity and control input of agent i, respectively, and $\mathcal{I}_m = \{1, 2, \dots, m\}$. Each first-order agent dynamics is given as follows:

$$\dot{x}_i(t) = u_i(t), \quad i \in \mathcal{I}_n - \mathcal{I}_m, \tag{2}$$

where $x_i \in \mathcal{R}$ and $u_i \in \mathcal{R}$ are the position and control input of agent i, respectively, and $\mathcal{I}_n = \{1, 2, \dots, n\}$. The dynamics of the leader is described as follows:

$$\dot{x}_0 = 0, \tag{3}$$

where $x_0 \in \mathcal{R}$ is the position of the leader. Let $[x_1(0), v_1(0), \dots, x_m(0), v_m(0), x_{m+1}(0), \dots, x_n(0)]^T$ be the initial condition of system (1)–(2).