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A triangle model of criminality

Juan C. Nuño[a,*], Miguel A. Herrero[b], Mario Primicerio[c]

[a] Departamento de Matemática Aplicada a los Recursos Naturales, E.T.S.I. Montes, Universidad Politécnica de Madrid, 28040 Madrid, Spain
[b] Departamento de Matemática Aplicada, Facultad de Ciencias Matemáticas, Universidad Complutense de Madrid, 28040 Madrid, Spain
[c] Dipartimento di Matematica "Ulisse Dini", Università degli Studi di Firenze, 55015 Firenze, Italy

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Abstract

This paper is concerned with a quantitative model describing the interaction of three sociological species, termed as owners, criminals and security guards, and denoted by X, Y and Z respectively. In our model, Y is a predator of the species X, and so is Z with respect to Y. Moreover, Z can also be thought of as a predator of X, since this last population is required to bear the costs of maintaining Z.

We propose a system of three ordinary differential equations to account for the time evolution of $X(t)$, $Y(t)$ and $Z(t)$ according to our previous assumptions. Out of the various parameters that appear in that system, we select two of them, denoted by H, and h, which are related with the efficiency of the security forces as a control parameter in our discussion. To begin with, we consider the case of large and constant owners population, which allows us to reduce (3)–(5) to a bidimensional system for $Y(t)$ and $Z(t)$. As a preliminary step, this situation is first discussed under the additional assumption that $Y(t) + Z(t)$ is constant. A bifurcation study is then performed in terms of H and h, which shows the key role played by the rate of casualties in Y and Z, that results particularly in a possible onset of bistability. When the previous restriction is dropped, we observe the appearance of oscillatory behaviours in the full two-dimensional system. We finally provide a exploratory study of the complete model (3)–(5), where a number of bifurcations appear as parameter H changes, and the corresponding solutions behaviours are described.

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Keywords: Criminality; Nonlinear dynamics; Sociological systems

1. Introduction

Crime has always been a serious concern in human societies. Indeed, references to criminal behaviour (and measures to check it) are well documented from the oldest extant records to the present (cf. Refs. [1,2,25,3,4]). While the concept of criminal (hence unacceptable) conduct largely differs among various cultures and historical periods, there is some general agreement in considering certain acts as criminal. In particular, this applies to the unauthorized

* Corresponding author. Tel.: +34 913367107; fax: +34 913363230.
E-mail addresses: juancarlos.nuno@upm.es (J.C. Nuño), Miguel_Herrero@mat.ucm.es (M.A. Herrero), primicer@math.unifi.it (M. Primicerio).

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exaction of resources belonging to a person or group of persons, legally considered to be their owners. This is the type of criminal behaviour addressed in this paper.

It is widely assumed that there is an enormous variability in crime rates, both in space and time [26]. Herein, we shall be concerned with some quantitative models that describe the evolution in time of three interacting populations. They will be referred to as owners, criminals and security guards, respectively. These are the three essential players in the so called *Routine Activity Theory* [5,6], one of the current conceptual frameworks being used in quantitative criminology (see also Ref. [28]). As concerns the interaction of these species, the following assumptions will be made:

A.1 Owners population (to be denoted as $X(t)$) are prey to the criminal population (to be denoted as $Y(t)$).

A.2 Security guard population (to be represented as $Z(t)$) not only acts as predator on the criminal population, but also on the owners', since the latter has to bear the cost of keeping the actual numbers of $Z(t)$.

A.3 A crime is committed whenever a criminal meets an owner in the absence of security surveillance. This statement is often referred to as the *Triangle Dogma* [7].

A.4 In any criminal event, two characteristic time scales can be distinguished. More precisely, a first period in which a criminal stalks prey is followed by a second stage in which that criminal is actively and exclusively engaged in committing that crime. Assuming this behaviour, the functional response of the criminal population $C_f(t)$ is given by Refs. [8,9]:

$$C_f(t) = \frac{kX(t)}{E + X(t)} Y(t) \tag{1}$$

for some constants $k \geq 0$ and $E \geq 0$.

A.5 The process of neutralisation of criminals by security forces follows a kinetics similar to that described in our previous hypothesis. More precisely, the rate of removal of criminals by the guards actions, $S_f(t)$, is given by:

$$S_f(t) = \frac{HY(t)}{D + Y(t)} Z(t) \tag{2}$$

for some $H \geq 0$ and $D \geq 0$. The parameter H will be termed as the *efficiency* of the security guards and will play a key role in the forthcoming study.

We next state the triangle model which incorporates assumptions (A.1)–(A.5). It reads as follows:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = r(N - X)(K - X) - \frac{kX}{E + X} Y - B\frac{Z}{X} \tag{3}$$

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = f\frac{SX}{E + X} Y - \frac{HY}{D + Y} Z - FY - GY^2 \tag{4}$$

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = g\frac{SX}{E + X} Y - h\frac{HY}{D + Y} Z - CZ. \tag{5}$$

Concerning Eqs. (3)–(5), some remarks are in order. To begin with, space-dependent properties are not included in this system. Moreover, (3)–(5) represent a continuum-based deterministic approach. This framework is particularly well suited to detecting changes in the dynamics of solutions associated with bifurcations arising when some control parameter reaches critical values, a strategy that will be exploited in the sequel. Discrete and stochastic effects are therefore excluded (or averaged out in a suitable manner). For a general discussion on the features of these different modelling approaches, the reader is referred to Ref. [10].

Eq. (3) asserts that, in the absence of criminals and guards, the owners population obeys a logistic-type dynamics with a growth rate $r > 0$ and a maximum capacity $N > 0$. The parameter $K < N$ is the optimal reachable population and so, $X(t) \to K$ as $t \to \infty$ if either $0 \leq X \leq K$ or $K \leq X < N$. Note that in this formulation the extinction state $X = 0$ is not a steady state. The presence of criminals hinders the growth of X at a rate given by (1), as described by the second term on the right hand side of (3). In its turn, the existence of security forces represents a cost which is shared by all owners in the manner described by the latest term on the right hand side of (3), with $B > 0$ there.

Eq. (4) describes the evolution in time of the criminal population $Y(t)$. It is therein assumed that $Y(t)$ increases according to the criminal rate (1) with a proportionality constant $f > 0$. On the other hand, $Y(t)$ decreases under the