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Gallagher index for sociophysical models

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Abstract

The use of physical formalism and methods as a tool for the analysis of sociological problems became popular since the papers of Galam or Stauffer, followed by the authors of some interesting models, based often on the study of cellular automata evolution. In this paper we are going to deduce some characteristics concerning the results of elections on the basis of the multi-opinion Sznajd model. The values of interest are: Gallagher index, measuring the disproportionality of elections; the efficiency of election process measured with the possibility of government creation and government stability. The results show that it is possible to obtain some well known effects even using quite a simple model of social behaviour and that different methods of votes counting respond to different needs formulated as the aim of elections.

Keywords: Sociophysics; Sznajd model

1. Introduction

The extension of computational possibilities offered by contemporary computers parallelly with the progress of calculational methods of critical phenomena theory have made it possible recently to try to study phenomena related to processes which are, in their full theoretical description, far away from the physical background. I mean here such problems as: forest fires [2,16], the cohabitation and struggle of various species in the same territory (usually within the frame of predator–prey model) [14] or the opinion formation in the community [9,12].

In this paper we are going to apply the modified Sznajd model [9] to the problem of analysis of election results. In the seminal sociophysical papers one mainly uses the word opinion as a description of studied state of individuals or the whole system. Here we are interested in the effect of simulation on the final distribution of mandates, so the words opinion and candidate/party will be used alternatively.

Sociologists usually underline two main functions of elections: to reproduce as exactly as possible the distribution of opinions observed in the population in the legislative bodies and to create a condition to form a stable government. The disproportionality of elections is measured using the so-called Gallagher index (Least Squares index) [6]:

$$LSq = \sqrt{\frac{1}{2}\sum_i (V_i - S_i)^2}, \tag{1}$$

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Table 1
Typical values of Gallagher index for some exemplary cases: specific countries and systems [3] [private calculations]

Country	System	LSq
Poland	Proportional	6.96
United Kingdom	Majority	5.43
New Zealand	Mixed	1.11
Averaged		
One-seat constituencies		12.93
Multi-seat constituencies		6.08
One constituency for whole country		2.27

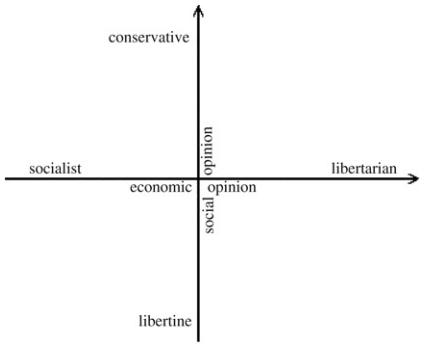

Fig. 1. Four possible options.

where summation is taken over all parties, V_i is the percentage of votes spend on the given party and S_i is the percentage of seats won by it. The typical values of the Gallagher index are shown for a few countries (with their electoral system) as well as averaged over countries with the same system in Table 1.

It should be mentioned that the Gallagher index presented above is certainly not the only attempt to measure representation disproportionality. Since the first idea presented by Rae in 1967 there were proposed few tens of different indices which had the good description of this factor as an aim. They use various mathematical approaches, like: different measures of deviations (eg. Rae, Loosemore-Hanby), analysis of regression (Cox-Shugart), χ^2 factor (Nagel) or well known from economical sciences, based on distribuant analysis, Gini index. The comprehensive list may be found eg. in: Refs. [8,15].

2. Models

In the calculations there was used the model being the extension of the Sznajd model [9] into larger than two opinions possible. This is obviously caused by the need to better reproduce the diversity of opinions in the community. These opinions are however not exactly independent. We use the model in which the individual's choices are considered in respect to two factors: The economical and social identification. Everyone can be described as more libertarian or more socialist from an economic point of view and more conservative or more libertine from the social point of view [10,1]. This model is schematically shown in Fig. 1. We will not discuss here the sociological problem of the naming convention for all presented ideas but only stress the effect of such a "two-dimensional" opinion scheme on the final result of elections.

Notice that the division into four opinions don't influence the way of updating the system. It is rather similar to the Potts model of magnetism. It would be however important in the process of votes counting, described later.

During the simulation there were no constraints put on the transition possibilities between the states characterizing the given cell. Every individual can change the opinion drastically even between the opposite quarters of Fig. 1.

The simulation was performed for two cases. The first one is the one-dimensional chain, for which the update rule is exactly the same as in the original Sznajd paper (obviously, except for the increase of number of options). It means