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A dynamic model for traffic network flow

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Abstract

Concerning the link properties in traffic networks, we introduce a dynamic equation of road flow into each link, and thereby propose a dynamic model for network flow. Using this model, we investigate the evolutions of inflow, outflow and flow on each link caused by a small perturbation of the network inflow under different route choice rules. Numerical results show that the dynamic model can reasonably capture the basic characteristics of network flow.

Keywords: Dynamic model; Network traffic flow; Inflow; Outflow

1. Introduction

Since the 1950s, many models have been developed to study complex traffic phenomena. Chowdhury [1] in detail introduced the existing traffic flow models (including kinematics models, dynamics models, gas-based kinematics models, car-following models and cellular automaton models), which can perfectly reproduce stop-and-go traffic, phase transitions, local clusters and various traffic waves. In addition, Gipps [2] presented the Gipps model, and Spyropoulou [3] recently investigated its mathematical properties. However, these models cannot directly be used to study network flow. On the other hand, the network models mainly concentrate on some macro issues (e.g. route choice) [4] but cannot depict the evolution of the flow and the density of each link. Therefore, developing a model which can deal with the macro properties of network flow and the drivers' micro behaviors of each link simultaneously has become an important topic in the field of traffic science.

Recently, some scholars adopted a cellular automaton model and a differential equation model to study network flow, and they obtained some important results [5–12]. Recently, some network flow simulation programs such as AIMSUN and DRACULA have been developed. In this paper, we develop a dynamic model for traffic network flow, and numerically test the model in a network with one OD pair connected by two routes. A small perturbation of the network inflow is set and the resultant evolutions of inflow, outflow and flow on each link are obtained. It is found that the dynamic model can reasonably capture the basic characteristics of network flow.

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2. Model

Lighthill and Whitham [13] and Richard [14] independently developed a first-order continuum traffic model (LWR model for short), which can be written as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_e)}{\partial x} = s(x, t), \tag{1}$$

where ρ is the traffic density, v_e the equilibrium velocity, $s(x, t)$ the traffic generation rate, and x and t the space and time variables respectively. If the freeway has no ramp, then $s(x, t) = 0$, and $s(x, t) \neq 0$ otherwise. In addition, the equilibrium velocity $v_e(\rho)$ is a decrease function of the density ρ and the equilibrium flow $\rho v_e(\rho)$ is a concave function of the density ρ. The LWR model can perfectly reveal the formation, propagation and evolution of shocks [15], but does not faithfully describe the non-equilibrium traffic flow because the velocity cannot deviate from the equilibrium velocity $v_e(\rho)$. For these reasons, many scholars have later developed various high-order models, which can be classified into the density-gradient (DG) model and speed-gradient (SG) model. The DG model can be written as follows [16–20]:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{v_e - v}{\tau} - \frac{c^2(\rho)}{\rho}\frac{\partial \rho}{\partial x} + \mu\frac{\partial^2 v}{\partial x^2}, \tag{2}$$

where τ is the reactive time, $c(\rho) > 0$ the sonic speed, and μ the viscous coefficient. The DG model can capture the basic properties of non-equilibrium flow, but may produce backward motion under some given condition [21]. Jiang et al. [22,23] recently presented an SG model:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{v_e - v}{\tau} + c_0\frac{\partial v}{\partial x}, \tag{3}$$

where the constant $c_0 > 0$ is the propagating speed of small perturbation. They proved that this model never gives backward motion and can reproduce such non-equilibrium phenomena as stop-and-go, local clusters and ghost jams. However, all the above models can only be used to describe road flow, rather than network flow.

In fact, it is very easy to extend the classical LWR model to the case of network flow. The extended LWR model can be written as follows:

$$\frac{\partial \rho_k}{\partial t} + \frac{\partial (\rho_k v_{ke})}{\partial x_k} = s_k(x_k, t), \quad k = 1, 2, \ldots, N, \tag{4}$$

where ρ_k and v_{ke} are respectively the traffic density and the equilibrium velocity of link k, $s_k(x_k, t)$ is the flow generation rate of link k, and N is the number of links in the network. In addition, the equilibrium velocity $v_{ke}(\rho_k)$ is a decrease function of the density ρ_k, and the equilibrium flow $\rho_k v_{ke}(\rho_k)$ is a concave function of the density ρ_k. Note that $s_k(x_k, t)$ in Eq. (4) is different from $s(x, t)$ in Eq. (1). It has the following form:

$$s_k(x_k, t) = \begin{cases} \text{the inflow of the link } k, & \text{if } x_k = 0 \\ 0, & \text{if } 0 < x_k < L_k \\ \text{the outflow of the link } k, & \text{if } x_k = L_k, \end{cases} \tag{5}$$

where L_k is the length of link k.

From Fig. 1, we can see that link k is independent of its adjacent links when $0 < x_k < L_k$, so we can directly introduce the full velocity difference (FVD) equation [22,23] into Eq. (4) and obtain a dynamic model for traffic network flow, i.e.

$$\begin{cases} \dfrac{\partial \rho_k}{\partial t} + \dfrac{\partial (\rho_k v_k)}{\partial x_k} = s_k(x_k, t) \\ \dfrac{\partial v_k}{\partial t} + v_k\dfrac{\partial v_k}{\partial x_k} = \dfrac{v_{ke} - v_k}{\tau_k} + c_k\dfrac{\partial v_k}{\partial x_k} \end{cases}, \quad k = 1, 2, \ldots, L, \tag{6}$$

where τ_k and c_k are the reaction time and the propagation speed of a small perturbation on link k, respectively. In general, τ_k and c_k are functions of density ρ_k. For simplicity, we here let them be constant, i.e., $\tau_k = \tau$ and $c_k = c_0$.