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The adiabatic static linear response function in nonextensive statistical mechanics

Guo Ran, Du Jiulin *

Department of Physics, School of Science, Tianjin University, Tianjin 300072, China

HIGHLIGHTS

- The difference between the three generations of the energy constraints is discussed in nonextensive statistical mechanics.
- The isothermal static linear response function is revisited using the third generation of the energy constraint.
- The adiabatic static linear response function is derived in nonextensive statistical mechanics.
- The relation between the isothermal and the adiabatic response function is presented.

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ABSTRACT

We analyze the difference between the three generations of the energy constraint in nonextensive statistical mechanics. Using the third generation of the energy constraint, we revise the isothermal static linear response function and then derive the static linear response function under the adiabatic condition. We present the relationship between the isothermal and adiabatic linear response functions.

1. Introduction

The power-law distributions widely exist in the complex systems in nature and society, and have attracted considerable attention in recent years. In many different fields of scientific research, the power-law distributions have been frequently observed and studied, such as astronomy and astrophysics [1–6], plasma physics and space science [7–16], chemistry and life science [17–24]. Because the description of power-law distributions is beyond the scope of the traditional theory governed by Boltzmann–Gibbs (BG) statistics, a new statistical theory, nonextensive statistical mechanics (NSM) which is as a generalization of BG statistics [25], has been developed for the study of power-law distributions. Correspondingly, the stochastic dynamical theory for the power-law distributions has also been in progress [26–28]. We know that in certain circumstances, the power-law distributions can represent the nonequilibrium stationary-state of a complex system.

In nonequilibrium statistical mechanics, the linear response theory of the stationary state has been a powerful tool for the study of the response of the system affected under a weak external force [29]. The study of the linear response theory in NSM therefore became an interesting question. Chame et al. studied the static linear response in NSM under an isothermal condition [30], and then Rajagopal worked the isothermal dynamical linear response theory [31]. In these works they employed the first or second generation of the energy constraint in NSM which may be not suitable for today's view. What

* Corresponding author.
E-mail addresses: jiulindu@aliyun.com, jldu@tju.edu.cn (J. Du).

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is more, they both worked only under the isothermal condition, and thus the linear response theory in NSM under the adiabatic condition is still unknown. The purpose of this work is to revisit the isothermal linear response theory using the third generation of the energy constraint in NSM and then to study the linear response theory under the adiabatic condition.

The paper is organized as follows. In Section 2, we will present a brief review on the three generations of the energy constraint in NSM and the differences between them. In Section 3, the isothermal linear response function will be revised under the third generation of the energy constraint. In Section 4, we will work out the linear response function under the adiabatic condition and then discuss the relation between the response functions under the two conditions. Finally in Section 5, we give the conclusion.

2. A brief review of the three generations of energy constraints in NSM

In the quantum theory, Tsallis entropy can be generally defined [25] as

$$S_q = k_B \frac{1 - Tr\hat{\rho}^q}{q-1}, \tag{1}$$

where k_B is the Boltzmann constant, $\hat{\rho}$ is the density matrix, and $q \neq 1$ is the nonextensive parameter. Maximizing Tsallis entropy subject to the normalization condition and the choice of the energy constraint, one could obtain the density matrix and define the q-expectation value for a physical quantity. With the development of NSM, three generations of the energy constraint were studied and appointed [25]. The first one is traditional,

$$U_q^{(1)} = Tr\left(\hat{H}\hat{\rho}\right), \tag{2}$$

the second one is q-dependent,

$$U_q^{(2)} = Tr\left(\hat{H}\hat{\rho}^q\right), \tag{3}$$

and the third one is q-normalized,

$$U_q^{(3)} = \frac{Tr\left(\hat{H}\hat{\rho}^q\right)}{Tr\hat{\rho}^q}, \tag{4}$$

where $\hat{H}$ is Hamiltonian of the system, and the superscripts (1), (2) and (3) distinguish the three energy constraints. With the three generations of the energy constraints, one can write the density matrices as a unified form [32],

$$\hat{\rho} = \frac{\left[1-(1-q^*)\,\beta^*\hat{H}\right]^{\frac{1}{1-q^*}}}{Tr\left[1-(1-q^*)\,\beta^*\hat{H}\right]^{\frac{1}{1-q^*}}}. \tag{5}$$

And they are equivalent to each other using the following parameter transformations, for the first constraint equation (2),

$$q^* = 2 - q^{(1)}, \tag{6}$$

$$\beta^* = \frac{\beta^{(1)}}{q^{(1)}Tr\hat{\rho}^{q^{(1)}} + \left(q^{(1)}-1\right)\beta^{(1)}U_q^{(1)}}, \tag{7}$$

for the second constraint equation (3),

$$q^* = q^{(2)}, \tag{8}$$

$$\beta^* = \beta^{(2)}, \tag{9}$$

and for the third constraint equation (4),

$$q^* = q^{(3)}, \tag{10}$$

$$\beta^* = \frac{\beta^{(3)}}{Tr\hat{\rho}^{q^{(3)}} + \left(1-q^{(3)}\right)\beta^{(3)}U_q^{(3)}}, \tag{11}$$

where $\beta^{(i)}$ with $i = 1, 2, 3$ corresponds to the Lagrange multipliers for the energy constraint equations (2)–(4) respectively.