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Suspended sediment concentration distribution using Tsallis entropy

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HIGHLIGHTS

- Tsallis entropy is employed for deriving the distribution of sediment concentration.
- The entropy based sediment concentration distribution is close to that from random walk model.
- The Tsallis entropy parameter is equivalent to nonlineary index of random walk model.

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ABSTRACT

Assuming sediment movement in channel flow as steady, Tsallis entropy is employed for deriving the distribution of sediment concentration. This distribution is found to be analogous to the distribution obtained from a random walk model. The sediment concentration distribution parameters are obtained from physical considerations. With these parameters, the distribution is tested using experimental observations and its sensitivity is evaluated. The agreement between entropy-based distribution and laboratory observations is found to be close.

1. Introduction

Consider a three-dimensional fluid medium in which flow is unsteady and nonuniform, that is, the velocity u is varying in all three directions x, y, and z, as well as in time t, $u(x, y, z; t)$. Here x represents the horizontal direction, y the vertical direction, and z the transverse direction. Let sediment be released into a channel from a single source. It has been shown [1,2] that the movement of sediment particles follows a "random walk". Then the position the sediment particle occupies during its movement can be considered as a random variable having a probability density function (pdf), $f(x, y, z)$; the pdf describes the random walk. This suggests that there is potential for employing entropy in dealing with sediment movement. In order to simplify the probabilistic treatment of sediment movement using the entropy theory, it is assumed that the flow is steady (i.e., u is independent of t) and so is sediment movement; and that the pdf does not vary in the longitudinal (x) and transverse (z) directions; thus, $f(x, y, z) = f(y)$ and y can be taken as the distance the particle travels. The objective here is to derive the sediment concentration distribution using the Tsallis entropy [3] that requires the derivation of pdf $f(y)$.

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2. Methodology for deriving sediment concentration

The methodology for deriving suspended sediment concentration distribution using the Tsallis entropy entails the following steps: (1) derivation of the probability density function of distance sediment particles travel in the flow, (2) determination of density function parameters from sediment hydraulics, (3) derivation of sediment concentration distribution, (4) probability distribution of sediment concentration, and (5) Tsallis entropy of sediment concentration.

2.1. Probability distribution of travel distance of sediment

Let the flow depth in the channel be denoted by D. Then $0 \le y \le D$. The Tsallis entropy for y or $f(y)$, $H[f(y)]$ or $H(y)$ can be written as:

$$H[f(y)] = H(y) = \frac{1}{m-1}\int_0^D f(y)\{1-[f(y)]^{m-1}\}\mathrm{d}y \tag{1}$$

where m is the Tsallis entropy index. The pdf of y is derived by maximizing the Tsallis entropy subject to the following constraints:

$$\int_0^D f(y)\mathrm{d}y = 1 \tag{2}$$

$$\int_0^D yf(y)\mathrm{d}y = E[y] = \bar{y} \tag{3}$$

where E is the expectation operator and $\bar{y}$ is mean value of y. For maximizing the Tsallis entropy given by Eq. (1), subject to Eqs. (2) and (3), the method of Lagrange multipliers is employed here, where the Lagrangian L can be written as

$$L = \frac{1}{m-1}\int_0^D f(y)\{1-[f(y)]^{m-1}\}\mathrm{d}y - \lambda_0\left[\int_0^D f(y)\mathrm{d}y - 1\right] - \lambda_1\left[\int_0^D yf(y)\mathrm{d}y - \bar{y}\right] \tag{4}$$

in which λ_0 and λ_1 are the Lagrange multipliers. Differentiating Eq. (4) with respect to f and equating the derivative to zero yield the maximum Tsallis entropy-based probability density function of y:

$$f(y) = m^{1/(1-m)}[1-(m-1)(\lambda_0+\lambda_1 y)]^{1/(m-1)}. \tag{5}$$

The Tsallis entropy of Eq. (5) can be written as

$$H(y) = \frac{1}{m-1} + \frac{m^{m/(1-m)}}{(2m-1)\lambda_1}\{[1-(m-1)(\lambda_0+\lambda_1 D)]^{(2m-1)/(m-1)} - [1-(m-1)\lambda_0]^{(2m-1)/(m-1)}\}. \tag{6}$$

The Lagrange multipliers λ_0 and λ_1 are now determined using Eqs. (2) and (3).

Substituting Eq. (5) in Eq. (2) yields

$$\int_0^D \left(\frac{1}{m}\right)^{1/(m-1)}[1-(m-1)(\lambda_0+\lambda_1 y)]^{1/(m-1)}\mathrm{d}y = 1. \tag{7}$$

Eq. (7) simplifies to

$$[1-(m-1)\lambda_0]^{m/(m-1)} - [1-(m-1)(\lambda_0+\lambda_1 D)]^{m/(m-1)} = m^{m/(m-1)}\lambda_1. \tag{8}$$

Substituting Eq. (5) into Eq. (3) results in

$$\int_0^D y\left\{\frac{1}{m}[1-(m-1)(\lambda_0+\lambda_1 y)]\right\}^{1/(m-1)}\mathrm{d}y = \bar{y}. \tag{9}$$

Upon integrating by parts, the solution of Eq. (9) is found to be:

$$\frac{1}{(2m-1)\lambda_1}\{[1-(m-1)\lambda_0]^{(2m-1)/(m-1)} - [1-(m-1)(\lambda_0+\lambda_1 D)]^{(2m-1)/(m-1)}\} - D[1-(m-1)(\lambda_0+\lambda_1 D)]^{(2m-1)/(m-1)} = m^{m/(m-1)}\lambda_1\bar{y}. \tag{10}$$

Eqs. (8) and (10) are solved simultaneously to determine the Lagrange multipliers λ_0 and λ_1. Their analytical solution is not tractable but the numerical solution is relatively straightforward.

It is seen that for given D and $\bar{y}$, the values of Lagrange multipliers depend on the value of m used, as shown for sample $D = 0.044$ m and $\bar{y} = 0.022$ m in Table 1. It can be seen from the table that λ_1 becomes small and less effective when m becomes large. With increasing m, λ_0 also becomes smaller. These values suggest that it is important that an appropriate