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A three-state kinetic agent-based model to analyze tax evasion dynamics

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HIGHLIGHTS

- Presence of three classes of agents: tax payers, tax evaders and undecided.
- Microscopic dynamics governed by pairwise interactions with quenched disorder.
- Punishment rules following the Zaklan model.
- Analysis of the steady-state fractions of three classes of agents.

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ABSTRACT

In this work we study the problem of tax evasion on a fully-connected population. For this purpose, we consider that the agents may be in three different states, namely honest tax payers, tax evaders and undecided, that are individuals in an intermediate class among honests and evaders. Every individual can change his/her state following a kinetic exchange opinion dynamics, where the agents interact by pairs with competitive negative (with probability q) and positive (with probability $1-q$) couplings, representing agreement/disagreement between pairs of agents. In addition, we consider the punishment rules of the Zaklan econophysics model, for which there is a probability p_a of an audit each agent is subject to in every period and a length of time k detected tax evaders remain honest. Our results suggest that below the critical point $q_c = 1/4$ of the opinion dynamics the compliance is high, and the punishment rules have a small effect in the population. On the other hand, for $q > q_c$ the tax evasion can be considerably reduced by the enforcement mechanism. We also discuss the impact of the presence of the undecided agents in the evolution of the system.

1. Introduction

In the recent years, the statistical physics techniques have been successfully applied in the description of socioeconomic phenomena. Among the studied problems we can cite opinion dynamics, language evolution, biological aging, dynamics of stock markets, earthquakes and many others [1–4]. These interdisciplinary topics are usually treated by means of computer simulations of agent-based models, which allow us to understand the emergence of collective phenomena in those systems.

A challenging interdisciplinary subject is tax evasion dynamics, which is an interesting practical topic to be studied because tax evasion remains to be a major predicament facing governments [5–7]. Models of tax evasion were firstly studied by economists [8–12], and more recently physicists became also interested in the subject [13–18] (for recent reviews, see

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Refs. [5,6]). Experimental evidence provided by Gachter suggests that tax payers tend to condition their decision regarding whether to pay taxes or not on the tax evasion decision of the members of their group [8]. In addition, Frey and Togler also provide empirical evidence on the relevance of conditional cooperation for tax morale [9]. Based on these ideas, Zaklan et al. recently proposed a model that has been attracted attention [13]. In the so-called Zaklan model, the dynamics of tax payers and tax evaders is analyzed by means of the two-dimensional Ising model at a given temperature T. In this model, each agent i may be in one of two possible states, namely $s_i = +1$ (honest) or $s_i = -1$ (cheater or tax evader). A transition $s_i \to -s_i$ (or a spin flip) is controlled by the "social temperature" T and also depends on the nearest neighbors' states of the agent (or spin) at site i. Thus, for low temperatures few spin flips occur and for high temperatures many spin flips occur. In other words, tax evaders have the greatest influence to turn honest citizens into tax evaders if they constitute a majority in the respective neighborhood. In addition, some punishment rules are applied: there is a probability p_a of an audit each agent is subject to in every period and a length of time k detected tax evaders remain honest [13]. In another work, the dynamics of the model was also controlled by another two-state model, namely the majority-vote model with noise [19], where the noise q plays the role of the temperature. In this case, similar results were found [16], suggesting that the results of the Zaklan model are robust.

In this work we study the tax evasion dynamics by means of a three-state agent-based model. The agents interact by pairs considering kinetic exchanges of their states, in a way that the pairwise couplings may be positive or negative. In addition, we apply the punishment rules of the Zaklan econophysics model. Our results suggest that above the critical point of the opinion dynamics the tax evasion can be considerably reduced by the enforcement rules. On the other hand, below the critical point the compliance is high, and the punishment rules have a small impact on the evasion.

This work is organized as follows. In Section 2 we present the microscopic rules that define the model, and the numerical results are discussed in Section 3. Finally, our conclusions are presented in Section 4.

2. Model

Our model is based on a kinetic exchange opinion model [20]. A population of N agents is defined on a fully-connected graph, i.e., each agent can interact with all others. In opposition to what occurs in the Zaklan model [13], for which the dynamics is governed by the Ising model (i.e., a two-state model), in our model each individual i ($i = 1, 2, \ldots, N$) carries one of three possible states or attitudes at a given time step t, represented by $s_i(t) = +1, -1$ or 0. The dynamic rules are defined following the opinion model of Ref. [20]. Each social interaction occurs between two given agents i and j, and we considered that j will influence i. First, this pair of agents (i, j) is randomly chosen. Then, the state of the agent i in the next time step $t + 1$ will be updated according to

$$s_i(t+1) = \mathrm{sgn}\left[s_i(t) + \mu_{ij}\, s_j(t)\right], \tag{1}$$

where the sign function is defined such that $\mathrm{sgn}(0) = 0$ and the interaction strengths $\{\mu_{ij}\}$ are quenched random variables given by the discrete bimodal probability distribution

$$F(\mu_{ij}) = q\delta(\mu_{ij} + 1) + (1 - q)\delta(\mu_{ij} - 1). \tag{2}$$

Notice that we considered that each agent can in principle interact with all other agents, i.e. there is no specific underlying topology for the structure of the interaction network. So the model can be viewed as an infinite dimension (or "mean field") Zaklan model. This is an almost realistic situation thanks to the modern social and communication networks.

First, let us elaborate upon the nature of the above-mentioned three states. The state $s_i = +1$ represents an honest tax payer, i.e., an individual 100% convinced of his/her honesty, who does not consider evasion. He/she is either habitually compliant or he/she is a recent evader who has become honest as a result of enforcement efforts or social norms. On the other hand, the state $s_i = -1$ represents a cheater, i.e, an individual who is an evading tax payer. Whether a tax payer continues to evade depends on both enforcement and the effect of social interactions.

Those two classes correspond to the ± 1 states of the standard Zaklan model [13]. In addition, we have considered a third state, $s_i = 0$, which can be interpreted as an *undecided* individual. However, notice that the above rules of the opinion dynamics [Eqs. (1) and (2)] impose that for an agent to shift from state $s = +1$ to $s = -1$ or vice-versa it must pass by the intermediate undecided state $s = 0$ [20]. Thus, an agent that is currently at the state $s = 0$ was an honest tax payer ($s = +1$) or a tax evader ($s = -1$) before. In the first case, the individual is an honest tax payer and, due to social interactions, he/she becomes a tax payer who is dissatisfied with the tax system, perhaps as a result of seeing others evading without being punished. He/she is not actively evading, but he/she might if the perceived benefits of doing so exceed the perceived costs. For this group, evasion is an option. On the other hand, the second possibility is that the agent is a tax evader and, due to social interactions, he/she stops temporarily the evasion because he/she wondered whether it is worth to evade. This agent is fickle and is not 100% convinced of his/her honesty, and thus he/she can become an honest tax payer ($s = +1$) or he/she can come back to the tax evader state ($s = -1$), depending on the next interactions with his/her social contacts. The above discussion will become more clear in the following, when we will discuss the interpretation of the competitive interactions μ_{ij}.

The pairwise couplings μ_{ij} in Eq. (2) may be either negative (with probability q) or positive (with probability $1 - q$), such that q represents the fraction of negative couplings [20,21]. The above process given by Eqs. (1) and (2) is repeated N times, which defines one time step in the dynamics. In addition to such basic dynamics of the model, after the N interactions we