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From the time series to the complex networks: The parametric natural visibility graph

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ABSTRACT

We present the modification of natural visibility graph (NVG) algorithm used for the mapping of the time series to the complex networks (graphs). We propose the parametric natural visibility graph (PNVG) algorithm. The PNVG consists of NVG links, which satisfy an additional constraint determined by a newly introduced continuous parameter—the view angle. The alteration of view angle modifies the PNVG and its properties such as the average node degree, average link length of the graph as well as cluster quantity of built graph, etc. We calculated and analyzed different PNVG properties depending on the view angle for different types of the time series such as the random (uncorrelated, correlated and fractal) and cardiac rhythm time series for healthy and ill patients. Investigation of different PNVG properties shows that the view angle gives a new approach to characterize the structure of the time series that are invisible in the conventional version of the algorithm. It is also shown that the PNVG approach allows us to distinguish, identify and describe in detail various time series.

1. Introduction

The idea to investigate the time series by mapping them to the complex networks (graphs) is very attractive. Two advanced research areas are combined under this approach: the methods of the nonlinear time series analysis [1–8] and the theory of the complex networks [9–15]. There is an opportunity to apply the rich, well-developed methods of the complex networks analysis to the investigation of the time series with a complicated structure, such as the fractal time series.

Currently there are several algorithms for mapping the time series to complex networks. For instance, it was suggested in Ref. [16] to build a network using the proximity of the coordinates in the Poincare section of the time series (see also Refs. [17–20]). Visibility mapping algorithms were proposed by *L. Lacasa et al.* in Refs. [21,22] for constructing a Natural Visibility Graph (NVG) [21] and Horizontal Visibility Graph (HVG) [22].

Let $\{x(t_i),\ i = 1 \dots N\}$ be a time series N of data, t_i are in natural temporal ordering. The NVG [21] is created by mapping of a time series of N data to a network (graph) of the N nodes. The link (i, j) belongs NVG if on the time series plot $x(t_k)$ for all t_k between t_i and t_j are below the line connecting $x(t_i)$ and $x(t_j)$ (see Fig. 1(a), (b)). The HVG algorithm [22] is similar to the NVG algorithm but has a modified mapping criterion. The link (i, j) exists in the HVG only if all $x(t_k)$ for t_k between t_i and t_j are less than both $x(t_i)$ and $x(t_j)$ (see Fig. 1(c), (d)).

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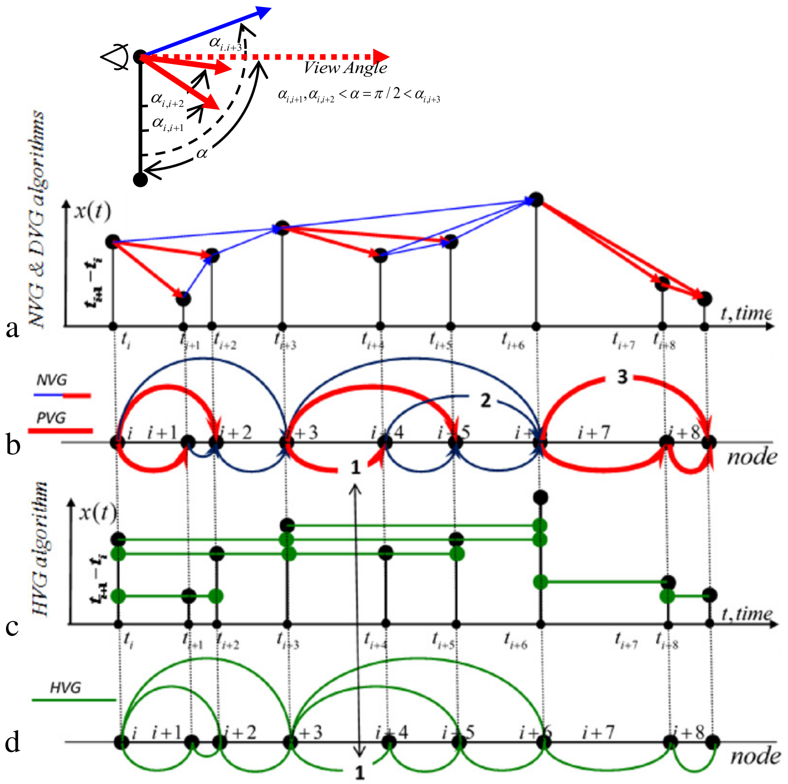

Fig. 1. Illustration of the Visibility Graph algorithms. Time series plot for NVG and PNVG algorithms (a) and HVG algorithm (c), corresponding graphs (b) and (d). On the time axis (a) and (c) events $t_i \dots t_{i+8}$ are marked. Upper left—the PNVG link selection criterion for view angle $\alpha = \pi/2$ applied to node $x(t_i)$. Lines t_i, t_{i+1} and t_i, t_{i+2} with $\alpha_{i,i+1}$ and $\alpha_{i,i+2}$ (thick lines, red online) belongs to the PNVG($\pi/2$), the link with $\alpha_{i,i+3}$ (thin lines, blue online) does not belong to the PNVG($\pi/2$). The NVG (b) consists of thick (red online) and thin (blue online) links. The PNVG($\pi/2$) (b) consists only of thick (red online) links. On the bottom (d) HVG links (green online) are shown. Marked links: 1—belongs to NVG, HVG and the PNVG($\pi/2$), because its view angle is less than $\pi/2$, 2—belongs only to the NVG, because its angle is greater than $\pi/2$, 3—the link that belongs to the NVG and the PNVG($\pi/2$) but not to the HVG. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

NVG and HVG are the connected graphs, and the HVG is a subgraph of the NVG. The HVG and the NVG are invariant under affine transformations of time series plot [21,22].

The use of the NVG and the HVG algorithms allows us to describe and explore the time series of complex structure associated with a variety of phenomena: fluctuations of turbulent flows [23], stock market indices [24], human heartbeat dynamics [25,26], stochastic and chaotic series [27–30] and others [31–37].

In this paper we introduce the parameter α which we call "View angle" and describe the algorithm to create the Parametric Natural Visibility Graph (PNVG) whose mapping criteria depends on view angle α. The PNVG algorithm consists of three major steps: first we build NVG [21], then we set the natural temporal direction to each NVG link and set the weight equal to the angle between link and downward directions (see Fig. 1(a)) and finally we construct PNVG from NVG links which has weight (angle) less than the given view angle α. The PNVG is a directed subgraph of the underlying NVG but PNVG is not necessarily a connected graph.

Actually, the ability to change arbitrarily view angle α adds the word "parametric" to the name of the algorithm. We will also use the abbreviation PNVG(α). The PNVG algorithm creates a new graph for each view angle α. In such a way we are able to investigate the properties of the PNVG(α) how they depend upon view angle α.

In the article, the mapping criterion for the Parametric Natural Visibility Graph algorithm is proposed and the relation between PNVG, NVG and HVG is discussed. Then we introduce and investigate the behavior of PNVG(α) properties: the average nodes degree, the average link length of the graph, and the number of clusters in the PNVG(α). Different artificial random time series are considered, i.e. with and without correlations and, more complicated, having fractal dimension too. At the end, we show an example of application of the PNVG algorithm to the heart beat *RR* intervals analysis and show the possibility to classify different types of heart failure.

2. Model and algorithm

We start the construction of the PNVG from a sequential time series $\{x(t_i),\ i = 1 \dots N\}$ of N data such as lighting event times or solar flare or earthquake magnitudes, etc.