Available online at www.sciencedirect.com
ScienceDirect
Physica A 387 (2008) 261–269
PHYSICA A
www.elsevier.com/locate/physa

Multifractal properties of Chinese stock market in Shanghai

Guoxiong Du[a,b,*], Xuanxi Ning[a]

[a] *College of Economics and Management, Nanjing University of Aeronautics and Astronautics, 29 Yudao Street, Nanjing, P.C. 210016, PR China*
[b] *Nanjing Institute of Industry and Technology, Xianlin College City, P.C. 210046, PR China*

Received 25 July 2006; received in revised form 23 July 2007
Available online 19 August 2007

Abstract

In this article, we apply three methods of multifractal analysis, partition function method, singular spectrum method and multifractal detrended fluctuation analysis method, to analyze the closing index fluctuations of Shanghai stock market during the past seven years. We have found that Shanghai stock market has weak multifractal features and there are long-range power-law correlations between index series. The shapes of singular spectrums do not change with time scales and their strengths weaken when the scales shorten. But when the orders of partition function increase, the strengths of multifractal increase, the singular spectrums become rougher and the general Hurst exponents decrease. These results provide solid and important values for further study on the dynamic mechanism of stock market price fluctuation.

Keywords: Multifractal analysis; Stock index fluctuation; Chinese stock market

1. Introduction

Fractal exists widely not only in nature and biology, such as the coastlines, the shapes of lightnings, the distributions of rivers, the structures of leaves, lungs and brains, but also in physics and chemistry, such as crystallization, phase transition, electrolysis and chemical oscillation. It also exists in economics, such as the distributions of income, the fluctuations of foreign exchange rate and stock indices and so on. It reveals the scale invariance of irregular bodies in the nature [1].

The scale is the unit of measurement. Generally speaking, every thing has its characteristic scale, such as characteristic length and characteristic time. When we study the universe, we should use cosmic scales. When we study the particles of the matter, we should use the microscopic scales. However, within determined scales of some matters, the results of measurement do not vary with the scales when they are in dynamic process, that is, they do not have any characteristic scales but have scale invariance.

The elementary function which has the property of scale invariance is the power function, $f(\tau)\sim\tau^m$, where m is called fractal dimension in fractal theory, denoted by D_f. It is the characteristic exponent indicating the

*Corresponding author. Nanjing Institute of Industry and Technology, Xianlin College City, P.C. 210046, PR China. Fax: +86 25 85864010.
E-mail address: dugx@niit.edu.cn (G. Du).

0378-4371/$ - see front matter
doi:10.1016/j.physa.2007.08.024

irregularity of data. When the scale τ is enlarged or reduced λ times, the new function is λ^m times the original function, $f(\lambda\tau) \sim \lambda^m f(\tau)$, and the shape and nature of the new function do not change.

Recently, people have found that different local conditions can cause different characters and show singularities. Thus, a single characteristic exponent or fractal dimension that depends on the whole cannot describe the singularities completely. Therefore, the multifractal was put forward, which describes the different characters of different areas of the whole body, with a spectrum function or singular function. If the fractal dimension D_f does not change with the scale τ, it is called monofractal. The rescaled range analysis (R/S) [2,3] and the detrended fluctuation analysis (DFA) [4] are the methods for monofractal. If D_f is changed with τ, it is called the multifractal, which is one of the important analysis methods for a non-linear complex system.

So far, many researches have revealed that the price or return fluctuations of different stock markets have multifractal properties [5–10], not only the developed markets [5,6] but also the newly established markets [8,9]; not only the short-scale indices series, such as a minute or a day [6,10,15], but also long-scale indices series, such as a year [5,8,9]. The multifractal properties of the fluctuations of price or return in stock markets are very important for the risk management and risk forecast. In this paper, we deeply study the multifractal properties of Shanghai stock market in China according to the closing index series over the past seven years with different scales.

2. Methods of multifractal analysis

The main task of multifractal theory is about the distribution of scale or the structure and character of the spectrum function and then to analyze the dynamic process of the original system. There are three methods.

(1) *Partition function method*: Supposing $\{x_i\}$ $(i = 1, 2, \ldots, T)$ to be the closing index series, T is the length of the series. Dividing it into N parts with an equal length t according to $\varepsilon = t/T$ $(\varepsilon < 1)$. Let $P_j(\varepsilon)$ the mass probability or mass density of the jth part $(j = 1, 2, \ldots, N)$,

$$P_j(\varepsilon) = I_j(\varepsilon) \Big/ \sum_{j=1}^{N} I_j(\varepsilon), \tag{1}$$

where $I_j(\varepsilon)$ is the sum of the index in part j.

Defining the partition function $\chi_q(\varepsilon)$ as

$$\chi_q(\varepsilon) \equiv \sum_{j=1}^{N} P_j^q(\varepsilon) = \varepsilon^{\tau(q)}, \tag{2}$$

where q is an integer, $q \in (-\infty, \infty)$. Different q indicates the different roles in the partition function played by different parts. Thus we can study the fractal body through different areas with different levels. With a given ε and q, we obtain $\chi_q(\varepsilon)$. In empirical analysis, the slope of $\ln \chi_q(\varepsilon) - \ln \varepsilon$ plot is $\tau(q)$. $\tau(q)$ is called the mass exponent. If it is a linear function of q, the original series is monofractal. If it is a convex function of q or $\ln \chi_q(\varepsilon) - \ln \varepsilon$ plot is composed of a group of lines with different slopes, the original series is multifractal. q and $\tau(q)$ are a group of parameters describing the multifractal.

(2) *Singular spectrum method*: According to fractal physics, multifractal describes the characters of different levels when the fractal geometric body grows. So the object, such as geometric body, index series, exchange rate series and so on, is divided into N parts with equal length ε $(\varepsilon < 1)$. Suppose the growing probability of different growing interface areas is $P_i(\varepsilon)$. Different areas have different growing probabilities, indicated by a different exponent α,

$$P_i(\varepsilon) \propto \varepsilon^{\alpha}, \quad i = 1, 2, 3, \ldots, N. \tag{3}$$

If all the areas have the same α, it is monofractal. Otherwise, if different areas have different α, it is multifractal. Let the areas with same α form a subset. Because of $\varepsilon < 1$, the minimum α corresponds to the maximum probability subset and vice versa. Let $N_\alpha(\varepsilon)$ be the number of areas with same α; thus [11],

$$N_\alpha(\varepsilon) \propto \varepsilon^{-f(\alpha)} \quad (\varepsilon \to 0), \tag{4}$$