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A moment approach to non-Gaussian colored noise

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Abstract

The Langevin system subjected to non-Gaussian colored noise has been discussed, by using the second-order moment approach with two kinds of models for generating the noise. We have derived the effective differential equation (DE) for a variable x, from which the stationary probability distribution $P(x)$ has been calculated with the use of the Fokker–Planck equation. The result of $P(x)$ calculated by the moment method is compared to several expressions obtained by different methods such as the universal colored noise approximation (UCNA) [Jung and Hänggi, Phys. Rev. A 35 (1987) 4464] and the functional-integral method. It has been shown that our $P(x)$ is in good agreement with that of direct simulations (DSs). We have also discussed dynamical properties of the model with an external input, solving DEs in the moment method.

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Keywords: Non-Gaussian noise; Colored noise; Moment method

1. Introduction

Interesting, unconventional phenomena such as the stochastic resonance (SR) and the noise-induced phase transition are created by noise. Theoretical studies on noise in nonlinear dynamical systems have usually adopted Gaussian white (or colored) noise. In recent years, there is a growing interest in studying dynamical systems driven by non-Gaussian noise. This is motivated by the fact that non-Gaussian noise with random amplitudes following the power-law distribution is quite ubiquitous in natural phenomena. For example, experimental results for crayfish and rat skin offer strong indication that there could be non-Gaussian noise in these sensory systems [1,2]. A simple mechanism has been proposed to generate the non-Gaussian noise [3]. With the use of such a theoretical model, the SR induced by non-Gaussian colored noise has been investigated [4]. It has been shown that the peak in the signal-to-noise ratio (SNR) for non-Gaussian noise becomes broader than that for Gaussian noise. This result has been confirmed by an analog experiment [5].

Stochastic systems with non-Gaussian colored noise are originally expressed by the non-Markovian process. This problem is transformed into a Markovian one by extending the number of variables and equations. The

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Table 1
A comparison among various approaches to the model A [Eqs. (1) and (10)] yielding the effective differential equation given by $\dot{x} = F_{eff} + I_{eff} + \alpha_{eff}\eta(t)$, where $r_q = 2(2-q)/(5-3q)$ and $s_q = 1 + (q-1)(\tau/2\phi^2)F^2$ (see text)

F_{eff}	I_{eff}	α_{eff}	Method
F	I	$r_q\phi/(\sqrt{1-r_q\tau f_1})$	Moment[a]
F	–	$r_q\phi/(\sqrt{1-r_q\tau F'_s})$	FI-1[b]
$F/(1-r_q\tau F')$	$(I+\tau\dot{I})/(1-r_q\tau F')$	$r_q\phi/(1-r_q\tau F')$	UCNA[c]
$F/(1-s_q\tau F')$	–	$s_q\phi/(1-s_q\tau F')$	FI-2[d]

[a] The moment method.
[b] Functional-integral (FI-1) method of Ref. [9].
[c] UCNA calculation after Ref. [6,7].
[d] Functional-integral (FI-2) method of Ref. [4].

relevant Fokker–Planck equation (FPE) includes the probability distribution expressed in terms of multi-variables. We may transform this FPE for multivariate probability to the effective single-variable FPE, or obtain one-variable differential equation (DE) with the use of some approximation methods like the universal colored noise approximation (UCNA) [6,7] and the functional-integral methods [8,9]. The obtained results, however, do not agree each other, depending on the adopted approximations, as will be explained in Section 2.2 (Table 1). It is not easy to trace the origin of this discrepancy because of the complexity in adopted procedures. The purpose of the present paper is to discuss the non-Gaussian colored noise and to make a comparison among various methods, by employing the second-order moment method which is simple and transparent, and which is exact in the weak-noise limit.

The paper is organized as follows. We have applied the second-moment method to the Langevin model subjected to non-Gaussian colored noise which is generated by two kinds of models. In Section 2, non-Gaussian colored noise is generated by the specific function which was proposed by Borland [3] and which has been adopted in several studies [4,8,9]. In contrast, in Section 3, non-Gaussian colored noise is generated by multiplicative noise [10–14]. We derive the effective one-variable DE, from which the stationary distribution is calculated with the use of the FPE. A comparison among various methods generating the non-Gaussian colored noise is made is Section 4, where contributions from higher moments than the second moment are also discussed. The final Section 5 is devoted to our conclusion.

2. Models A_0 and A

2.1. Moment method

We have adopted the Langevin model subjected to non-Gaussian colored noise (ε) and Gaussian white noise ($\psi\xi$), as given by [3]

$$\dot{x} = F(x) + \varepsilon(t) + \psi\xi(t) + I(t), \tag{1}$$

$$\tau\dot{\varepsilon} = K(\varepsilon) + \phi\eta(t), \quad (\text{model } A_0), \tag{2}$$

with

$$K(\varepsilon) = -\frac{\varepsilon}{[1 + (q-1)(\tau/\phi^2)\varepsilon^2]}, \tag{3}$$

which is referred to as the model A_0. In Eqs. (1)–(3), $F(x)$ is an arbitrary function of x, $I(t)$ stands for an external input, q is a parameter expressing a departure from the Gaussian distribution which is realized for $q = 1$, τ denotes the characteristic time of colored noise, and η and ξ the zero-mean white noises with correlations: $\langle\eta(t)\eta(t')\rangle = \delta(t-t')$, $\langle\xi(t)\xi(t')\rangle = \delta(t-t')$ and $\langle\eta(t)\xi(t')\rangle = 0$.