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Generating synthetic time series from Bak–Sneppen co-evolution model mixtures

F. Petroni[a,*], M. Ausloos[a], G. Rotundo[b]

[a] GRAPES, B5, Sart-Tilman, B-4000 Liege, Belgium
[b] Faculty of Economics, University of Tuscia, Viterbo, Italy

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Abstract

The Bak–Sneppen model of co-evolution is used to derive synthetic time series with *a priori* specified fractal dimension (or Hurst exponent) through a mixing of processes in various lattice dimensions. Both theoretical and numerical analyses concern the avalanches at the critical threshold and provide a model for time series reconstruction that can be tested as an alternative to the classical fractional Brownian motion (fBm) because of differences in properties. New results on critical threshold and avalanche structure are obtained up to Euclidean dimension $d = 6$. The method involves a lattice-based structure and therefore is suitable for the application of parallel computing.

Keywords: SOC model; Time series; Fractal dimension; Evolution model; Fractional Brownian motion

1. Introduction

Fractal processes, long-term memory processes, mean reverting processes, etc. play a key role in the analysis of time series when concerning the identification of the most suitable generating process. Usually the proposed dynamics do not enter either in the detail of pseudo-random number generator or in the procedure of the creation of the correlation structure starting from uniformly distributed random variables, often just relying on standard software libraries. The aim of this paper is to set up a lattice-based structure and a co-evolution dynamics for the generation of synthetic time series with predefined fractal dimension, starting from the Bak–Sneppen (B&S) co-evolution model [1]. This approach should thus be considered as complementary to and different from the classical fractional Brownian motion (fBm) [2].

The original self-organized criticality model of Bak and Sneppen (B&S) [1,3,4] was developed to fulfill the characteristics of species distribution evolution, and it has been shown to be suitable also for modeling spatial and temporal correlation functions in earthquakes, landslides, and financial time series phenomena [5–8]. The original d-dimensional B&S model deals with L^d species that compete for their survival and occupy the nodes of a d-dimensional lattice. Each species is fully described by its fitness $f_i^d(t)$, $i = 1, \ldots, L^d$ drawn at time 0 from

*Corresponding author.
E-mail addresses: filippo.petroni@ulg.ac.be (F. Petroni), marcel.ausloos@ulg.ac.be (M. Ausloos).

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a uniform distribution in [0, 1]. At each time t the lowest fitness $f_i^d(t)$ and its $2d$ neighbors (assuming square, cubic, ..., lattice symmetry) in the d-dimensional lattice are replaced by random numbers taken from a uniform distribution. Therefore, a change in fitness of one species implies an evolution of neighbors as well: there is co-evolution. In the original model the fitness can represent either the living capability of the species or the population, or a barrier to be overcome. The replacement of the lowest value $f_i^d(t)$ by a random number can be interpreted as a dynamical correction to the worst fitness.

When $L \to \infty$, and for t large enough the system reaches a stationary phase in which almost all species have their fitness above a threshold f_c^d; these fitnesses are therefore uniformly distributed in $(f_c^d, 1)$ [9,10]. The B&S model critical threshold f_c^d values are reported in Ref. [11]. However due to the evolution dynamics, the phase is only metastable, since the lowest fitness is immediately replaced by another arbitrary one, and the evolutionary process is restarted again. It has been shown that the fitness dynamics is therefore characterized by *avalanches*, and the critical threshold f_c^d fluctuates with time in finite size systems [8,12–14]. At this stage it is of interest to stress that the B&S model has led to several definitions of avalanches [1,10,13]. One may also distinguish among other things between the size and the duration of avalanches. The avalanche size was first defined as the number of sites which are spatially connected as long as the avalanche exists—the duration of an avalanche in the original B&S model referring to the time of existence of such a cluster. Other definitions exist, e.g. the duration of an avalanche can refer to the time spent by the lowest $f_i^d(t)$ below f_c^d. In Refs. [11–13], the size s of avalanches is defined as its temporal duration, i.e., an avalanche of size s remains below the critical threshold for $s-1$ time steps. It has already been shown [10–13] that the s-distribution of avalanches follows a power law:

$$P(s) \propto s^{-\tau_d}. \tag{1}$$

Moreover, for $d \geqslant 4$ a mean field approximation holds and the probability of avalanche size obeys a common decay behavior with exponent $\tau_{B\&S} = \frac{3}{2}$ [9,11].

The present analysis investigates also critical thresholds f_c^d as well as the avalanche size distribution decay exponent τ_d of the average fitness in a d-dimensional B&S model in the manner of Li and Cai (LC) paper [12]. LC have shown how to calculate the critical threshold $\bar{f}_c^d$ of the average fitness, i.e., $\bar{f}^d(t) = (1/L^d)\sum_{i=1}^{L^d} f_i^d(t)$, from an approximation,

$$\lim_{L\to\infty} \bar{f}_c^d = \lim_{L\to\infty} \frac{1+f_c^d}{2}, \tag{2}$$

in the limit of a large number of agents L^d. LC obtained results for $d = 1, 2$; we extend their study up to $d = 6$.

One can also calculate the autocorrelation function of $\bar{f}^d(t)$ as a function of the time lag θ; through simulations we found that such a signal property behaves like

$$\langle \bar{f}^d(t)\bar{f}^d(t+\theta)\rangle \sim \theta^{-(2-2H_d)}, \tag{3}$$

where H_d is the Hurst exponent that can be usually obtained through a detrended fluctuation analysis (DFA) [15].

Another motivation for the present work stems from the following argument. It is worth remarking that the standard time series analysis techniques test for the suitability of fBm [2] to raw data through usually some Hurst exponent [16] or fractal dimension measure, often without considering the avalanche feature. The fBm shows symmetric degradation and recovery duration to the initial time value that can be estimated through the first return time measure. Therefore, models in the class of extremal dynamics models, like B&S-based models, can be [17] and are proposed in the present report as an alternative to the classical fBm.

Moreover, it is clear that the main feature of the B&S model dynamics, i.e., its spatial dynamics, opens the way to the implementation of the model on parallel computers. The present paper shows how to combine models in dimension d in order to have an *a priori* correlation decay and a subsequent avalanche behavior. The results are compared with the linear combination of fBm.

The paper is organized as follows. The next section reports both theoretical and numerical estimates of the critical threshold and of the avalanche decay parameter for dimensions up to $d = 6$, along the numerical technique proposed by LC. Section 3 presents results based on a linear combination of the B&S models with different dimensions. Section 4 serves as a conclusion suggesting how to use the above results in the framework of time series modeling.