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Risk evaluation with enhanced covariance matrix

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Abstract

We propose a route for the evaluation of risk based on a transformation of the covariance matrix. The approach uses a 'potential' or 'objective' function. This allows us to rescale data from different assets (or sources) such that each data set then has similar statistical properties in terms of their probability distributions. The method is tested using historical data from both the New York and Warsaw stock exchanges.

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1. Introduction

Optimization of portfolios has been much studied since the pioneering work of Markowitz [1,2], who proposed using the mean–variance as a route to portfolio optimization [1–16]. However, the basic construction of the portfolio has not changed much as a result. Computation of Sharp ratios [17,18] and the Markowitz analysis equate risk with the co-variance matrix. Portfolio allocations are then computed by maximizing a suitably constructed utility function [19–21]. Moreover, the approach taken by Markowitz and many other authors [1,2] is essentially only appropriate for stochastic processes that follow random walks and exhibit Gaussian distributions [3–5]. Many economists have sought to use other utility functions and invoke additional objectives [22,23] in which portfolio weights are computed via maximization of these different utility functionals. Others have introduced additional features of the probability distribution such as the third moment or skewness of the returns [22,23]. This builds in aspects of the deviation of the probability distribution from the Gaussian as well as the asymmetry. Introducing even a constant value for the skewness may yield more reliable portfolio weights than a calculation in which only the variance or second moment of the distribution is used and where the risk of extreme values is seriously underestimated. Similar comments could be made about the introduction of the kurtosis which is a first order route to addressing the issue of 'fat' tails.

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In recent years, a number of physicists have begun to study the effect of correlations on financial risk. Techniques based on random matrix theory developed and used in nuclear physics have been applied to reveal the linear dependencies between stock market data for both the US and UK markets [4,5]. More recently other workers including one of the present authors have used minimum spanning trees methods [24–26] for the same purpose. Spanning tree methods seem to yield results that are similar to those obtained using random matrix theory but with less effort and the use of less data in the sense that only a subset of the correlation matrix is actually used to construct the tree. The overall aim, in both cases, is to arrive at optimal diversified portfolios. One interesting result obtained in Ref. [25] was the identification of new classifications introduced in the FTSE index ahead of their formal introduction by the London authorities.

An important outcome of studies basing on Markowitz approach is the capital asset pricing model (CAPM) [10,27–29] that relates risk to correlations within the market portfolio [10,27–29]; of course the risk now is that all investments will collapse simultaneously. Furthermore, it is assumed that risk that achieves premiums in the long term should not be reducible otherwise arbitrage is possible [28]. This is essentially the arbitrage pricing theory (APT).

However, key issues remain unresolved. One weakness of CAPM and APT theories is that they assume efficiency in the proliferation of market information. In a real market not all investors have the same or complete information and arbitrage is possible. Merton [30] has discussed this and in so doing has extended CAPM theory to deal more effectively with small firms for which information is not always readily available.

Here we concern ourselves with a new approach to the exploitation of data sets for the computation of portfolio weights within a diversified portfolio. The method exploits the full character of the distribution function for each asset in the portfolio and seeks to maximize the impact of correlations. In the next section, we discuss the background to our approach and introduce the so-called *objective* function. Having established this we show how, from data, we can construct values for a renormalized objective function. These are then used in Section 3 to obtain both covariance matrices and weights for portfolios of stocks. The calculations are illustrated in Section 4 by examples from both the US and Warsaw stock exchanges. We also show how the approach modifies the underlying distribution of eigenvalues enhancing the correlations for larger values.

2. Objective function

Consider an asset, characterized by a price, $S(t)$ and return $x(t) = \ln S(t+1)/S(t)$. The objective function, $w(x)$ is defined in terms of the stationary probability distribution for returns, $P(x)$, viz:

$$P(x) = \frac{1}{Z} \mathrm{e}^{-w(x)/D}, \tag{1}$$

where Z is a normalization factor. Such functions are familiar to physicists and may be derived by minimizing a 'free energy' functional, $F(w(x))$, subject to constraints on the mean value of the objective function, viz:

$$F = \int_R \mathrm{d}x P(x) \left[\ln P(x) + \frac{w(x)}{D} - \lambda \right]. \tag{2}$$

Such a form for the probability distribution is also the outcome of a model that assumes x is governed by a generalized Markovian stochastic process of the form:

$$\dot{x}(t) = f(x) + g(x)\varepsilon(t). \tag{3}$$

The Gaussian process, ε, satisfies:

$$\begin{aligned} \langle \varepsilon(t)\varepsilon(t') \rangle &= D\delta(t - t'), \\ \langle \varepsilon(t) \rangle &= 0. \end{aligned} \tag{4}$$

For the moment we leave the form of the functions f and g unspecified except to say that they only depend on $x(t)$. The solution to such a stochastic process has been deduced elsewhere [31–33]. Adopting the Ito convention, the distribution function, $P(x, t)$, associated with the process is given by the Fokker