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Synchronization in complex delayed dynamical networks with impulsive effects

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Abstract

The present paper is mainly concerned with the issues of synchronization dynamics of complex delayed dynamical networks with impulsive effects. A general model of complex delayed dynamical networks with impulsive effects is formulated, which can well describe practical architectures of more realistic complex networks related to impulsive effects. Based on impulsive stability theory on delayed dynamical systems, some simple but less conservative criterion are derived for global synchronization of such dynamical network. It is shown that synchronization of the networks is heavily dependent on impulsive effects of connecting configuration in the networks. Furthermore, the theoretical results are applied to a typical SF network composing of impulsive coupled chaotic delayed Hopfield neural network nodes, and are also illustrated by numerical simulations.

Keywords: Complex dynamical networks; Impulsive effects; Time-delays; Global synchronization; Scale-free network; Chaotic delayed Hopfield neural network

1. Introduction

Complex networks have recently been the subject of considerable interest within the science and technology communities. Especially, the study of nonlinear dynamical behaviors in complex dynamical networks, as well as how network topological structure influences its dynamical behaviors, has currently become a strategic topic of great significant [1–3]. There has been increasing interest in the study of synchronization dynamics in large-scale networks composing of coupled chaotic dynamical systems. Because they can exhibit many interesting phenomena such as spatiotemporal chaos, auto waves, spiral waves and others. In addition, it has been found that synchronization of coupled dynamical systems has potential applications in many fields including secure communication, parallel image processing, neural networks, biological systems, information science, etc. (see Refs. [4–9] and references cited therein).

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In many evolutionary systems there are two common phenomena: delay effects and impulsive effects [10–15]. For example, a signal or influence traveling through a network often is associated with time delays, and this is very common in biological and physical systems [16–19]. On the other hand, many evolutionary processes, particularly some biological systems such as biological neural networks and bursting rhythm models in pathology, as well as frequency-modulated signal processing systems, and flying object motions, are characterized by abrupt changes of states at certain time instants. This is the familiar impulsive phenomena [13,20–23]. Therefore, the study of dynamical networks associated with time delays and impulsive effects is important for understanding the dynamical behaviors of the most real-world complex networks.

Recently, synchronization dynamics of complex delayed dynamical networks have been extensively studied [16–21,24–27]. Some synchronization criteria of the dynamical networks with coupling delays are derived for both delay-independent and delay-dependent stability of synchronization manifold [16–18]. Furthermore, Zhou and Chen investigated synchronization phenomena in large-scale complex delayed dynamical networks composing of coupled chaotic oscillators with different small-world (SW) and scale-free (SF) interactions as well as the effects of time delays [19]. In addition, some authors are concerned with the synchronization problems of coupled neural networks with time delays and impulsive effects, where each dynamical node of the networks is a delayed Hopfield neural networks or delayed cellular neural networks (CNNs) [20,22–30].

The present paper is mainly concerned with the issues of synchronization of complex delayed dynamical networks with impulsive effects from the views of dynamics and control. The main objective of this research is devoted to the introduction of impulsive effects into topological structure of the networks, and then studies its synchronization dynamics. Based on impulsive stability theory on delayed dynamical systems, some simple but less conservative criteria are derived for global synchronization of such dynamical network. It is shown that synchronization of the networks is heavily dependent on the impulsive effects of connecting configuration in the networks. Furthermore, the theoretical results are applied to a typical SF network composing of coupled chaotic delayed Hopfield neural network nodes with impulsive effects, and numerical simulations are given to illustrate the theoretical results.

The rest of paper is organized as follows. In Section 2, a general model of complex delayed dynamical networks with impulsive effects is presented, and then some necessary definitions and preliminary results are given. In Section 3, some simple yet generic criteria are derived for determining global synchronization of such dynamical network. Section 4 applies these results to study global synchronization of a typical SF network composing of coupled chaotic delayed Hopfield neural network nodes with impulsive effects, where numerical examples are given to verify and also visualize the theoretical results. Finally, some concluding remarks are given in Section 5.

2. Problem formulations and preliminaries

First, we formulate a model of N linearly coupled delayed dynamical networks with impulsive effects described by the following measure differential equations:

$$Dx_i(t) = f(t, x_i(t), x_i(t-\tau)) + c\sum_{j=1}^{N} b_{ij}\Gamma x_j(t)Dw_j(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

in which $x_i(t) = (x_{i1}(t), \ldots, x_{in}(t))^\top \in R^n$ are the state variables of the ith delayed dynamical node, $f: R \times R^n \times R^n \to R^n$ is continuously vector-valued function, the constant $c>0$ is the coupling strength, the operator D denotes the distribution derivative, the bounded variation functions w_i: $[t_0, +\infty) \to R$ are right-continuous on any compact subinterval of $[t_0, +\infty)$, Dw_i depicts the impulsive effects of connecting configuration in the coupled dynamical network (1).

Without loss of generality, we assume that

$$Dw_j = 1 + \sum_{k=1}^{+\infty} u_k\delta(t-t_k), \quad j = 1, 2, \ldots, N, \tag{2}$$

where the fixed impulsive moments t_k satisfy $t_{k-1} < t_k$ and $\lim_{k\to\infty} t_k = +\infty$, $\delta(t)$ is the Dirac function, u_k represents the strength of impulsive effects of connection between the jth dynamical node and the ith