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A new community-based evolving network model

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Abstract

In order to describe the community structure upon the dynamical evolution of complex networks, we propose a new community based evolving network (CBEN) model having increasing communities with preferential mechanisms of both community sizes and node degrees, whose cumulative distribution and raw distribution follow scale-invariant power-law distributions $P(S \geqslant s) \sim s^{-v}$ and $P(k) \sim k^{-\gamma}$ with exponents of $v \geqslant 1$ and $\gamma \in [2, +\infty)$, respectively. Besides, complex networks generated by the CBEN model are hierarchically structured, which cover the range from disassortative networks to assortative networks.

Keywords: Complex network; Community; Preferential growth; Scale free; Hierarchy

1. Introduction

In recent years, the study of complex networked systems [1–4], including the Internet, the World Wide Web (WWW), social networks, biological networks, etc., has been an attractive issue. Historically, the most distinctive statistical properties that networks seem to share led to a number of important progresses in our understanding of complex networks, such as the ubiquity of small-world pattern [5], and scale-invariant feature of power-law degree distribution $P(k) \sim k^{-\gamma}$ [6]. Another very important characteristic owned in many real networks is the presence of communities, which is argued to be the signature of the hierarchical nature in complex networks [7–9].

Generally speaking, communities are defined as subsets of nodes within which connections are denser while between which are much sparser [10]. Therefore, finding communities in a network plays a crucial role in understanding the internal structure and principle function of clusters, which has been the subject of discussions in various disciplines. Although the problem of community detection is very difficult, researchers have proposed some very effective algorithms to find communities in networks having complex topologies [11–15].

To study the effects of community structure on network properties and dynamics, a natural question is: "How can we construct an evolving network model in the community structure?" With efforts of many researchers, several models for communities in social and biological networks have been proposed [16,17].

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The networked seceder model was proposed to construct a community structured network that emerged as an effect of the agents personal rationales [16]. An evolving model with merging building blocks was proposed to mimic the community structure in social networks and modules in biological systems [17]. As the realization of community structure in the World Trade Web, it was found that the preferential attachment mechanism is also within the local economic cooperative regions [18,19], i.e., the so-called local world phenomenon [20]. Recently, an evolving network model with inner-community preferential attachment, which owns the power-law degree distribution of the fixed exponent 3 [21], was built.

It has been pointed out that, like degree distributions, the size distribution of communities follows the power-law scaling property [11–15]. Especially, it has been pointed out that, like node degrees, the preferential mechanism also exists in the growing evolution of communities. That is, when establishing new links between communities or adding a new node to an existing community, communities with larger sizes are selected with higher probabilities [22]. Therefore, to demonstrate this phenomenon of "richer get richer" in communities scale, we develop the mechanism of community size preferential attachment in our newly proposed evolving network model with increasing communities in this paper.

The rest of this paper is organized as follows: In Section 2, we propose a new community based evolving network (CBEN) model. The growing dynamics of the CBEN model, including the community size and degree distributions, is analyzed in Section 3, where their scaling exponents are analytically studied. After the verification of numerical experiments in Section 4, the whole paper is finally concluded in Section 5.

2. Model description

In our CBEN model, an undirected and unweighted network is initialized with $c_0 (c_0 > 1)$ communities, each of which has n_0 fully connected nodes. There are $c_0(c_0-1)/2$ inter-community links to make c_0 communities fully connected. In each community, the node to which inter-links connect is selected at random. The model is evolved with the following scheme.

2.1. Growth

At each time step, a new community containing n_0 fully connected nodes is added with probability p, and we randomly choose one node in the new community to connect m nodes in other existing communities following the preferential attachment mechanisms.

A new node is added with probability $1-p$ at each time step. First, it chooses which community to add into, and then it connects m already existing nodes in the network. Each one of those m nodes is chosen from its own community with probability q and from other communities with probability $1-q$. Here q is assumed to be close to 1, or at least larger than 0.5, to guarantee that the connections of nodes within communities are denser than those between them.

Please note that when a new node chooses an inter-community neighbor, it firstly chooses a community and then chooses one node from that community. Although one certain community could be chosen for more than one time, multiple connections to one and the same node are not permitted.

2.2. Preferential attachment rules

(a) *Community size preferential attachment*: When a new node chooses an existing community to add (or chooses another community from which to get an inter-community neighbor), we assume the probability $\Pi(S_i)$ that it will choose community i depends on the size S_i of community i, such that

$$\prod(S_i) = \frac{S_i}{\sum_k S_k}. \tag{1}$$

(b) *Degree preferential attachment*: When choosing a neighbor, a new node firstly chooses a community i, which maybe its own community or another community according to Eq. (1), and then connects with one node in it. We assume the probability $\Pi(K_{ij})$ that the new node will connect to node j in community i is