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Analysis of diffusion and trapping efficiency for random walks on non-fractal scale-free trees

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HIGHLIGHTS

- Calculating the analytic solutions of mean trapping time for any target node.
- Calculating the analytic solutions of mean diffusing time for any source node.
- Comparing the trapping and diffusion efficiency among all nodes of the network.
- The trap's position has great effect on the trapping efficiency.
- The starting position almost has no effect on diffusion efficiency.

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ABSTRACT

In this paper, the discrete random walks on the recursive non-fractal scale-free trees (NFSFT) are studied, and a kind of method to calculate the analytic solutions of the mean first-passage time (MFPT) for any pair of nodes, the mean trapping time (MTT) for any target node and mean diffusing time (MDT) for any starting node are proposed. Furthermore, we compare the trapping efficiency and diffusion efficiency between any two nodes of NFSFT by using the MTT and the MDT as the measures of trapping efficiency and diffusion efficiency respectively, and find the best (or worst) trapping sites and the best (or worst) diffusion sites. The results show that the two hubs of NFSFT are not only the best trapping site but also the worst diffusion site, and that the nodes which are the farthest nodes from the two hubs are not only the worst trapping sites but also the best diffusion sites. Furthermore, we find that the ratio between the maximum and minimum of MTT grows logarithmically with network order, but the ratio between the maximum and minimum of MDT is almost equal to 1. The results imply that the trap's position has great effect on the trapping efficiency, but the position of starting node has little effect on diffusion efficiency. Finally, the simulation for random walks on NFSFT is done, and it is consistent with the derived results.

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1. Introduction

Many materials encountered in nature exhibit fractal scaling [1–4] and many problems in physics and chemistry are related to random walks on fractal structures [5,6]. Therefore, random walks on fractal media have attracted a lot of interest in the past few years [7–13].

The quantity we are interested in is the mean first-passage time (MFPT) $F(x, y)$, which is the expected number of steps before node y is visited, starting from node x. It is a quantitative indicator to characterize the transport efficiency and many other quantities can be expressed in terms of it. For example, one can define the mean trapping time (MTT) for node y by

$$T_y = \frac{1}{N-1} \sum_{x \in \Omega, x \neq y} F(x, y), \tag{1}$$

and the mean diffusing time (MDT) for node x by

$$D_x = \frac{1}{N-1} \sum_{y \in \Omega, y \neq x} F(x, y), \tag{2}$$

where Ω denotes the node set and N is the total number of nodes. Both the MTT and MDT vary with the position of node and they can be used as the measures of trapping efficiency and diffusion efficiency for network nodes respectively. Comparing the MTT and MDT among all the nodes, we can find the effects of node position on the trapping efficiency and diffusion efficiency. The nodes which have the minimum MTT (or the maximum MTT) are best (or worst) trapping sites and the nodes which have the minimum MDT (or maximum MDT) are the best (or worst) diffusion sites.

Although it is difficult to derive analytic solutions of MFPT, MTT and MDT on general fractal media, it can be exactly studied for deterministic fractals (or networks). In the past several years, a lot of endeavors have been devoted to study MFPT on different deterministic networks [13–19]. The MTT for some special nodes were obtained for different networks such as Sierpinski gaskets [16], Apollonian network [20], pseudofractal scale-free web [21], deterministic scale-free graph [22] and some special trees [23–27]. The MDT for some special nodes were obtained for exponential treelike networks [28], scale-free Koch networks [29] and deterministic scale-free graph [30]. There were also some works focusing on global mean first-passage time (GMFPT), i.e., the average of MFPTs over all pairs of nodes. These results were obtain for some special trees [24–26,31,32] and dual Sierpinski gaskets [33].

However, the previous results of MTT and MDT are only restricted to some special nodes for the above networks and we cannot compare trapping efficiency and diffusing efficiency among all the network nodes. It is still difficult to derive the analytic solutions of the MTT for any target node and the MDT for any source node in these networks. It is also difficult to derive the analytic solutions of MFPT for any pair of nodes.

As for the recursive non-fractal scale-free trees (NFSFT), the MTT for the hub node and the GMFPT were obtained [34]. The MTT for some low-generation nodes can also be derived due to the methods of Ref. [35]. But the analytic calculations of MFPT for any pair of nodes, the MTT for any target node and the MDT for any source node are still unresolved.

In this paper, we first express the MFPT, MTT and MDT for random walks on trees in terms of some quantities evolved from the shortest path lengths based on the relationship between random walks and electrical networks [36,37]. Then we propose methods to calculate these quantities of NFSFT based on its self-similar structure. Therefore, the analytic solutions of the MFPT for any pair of nodes, the MTT for any target node and MDT for any starting node are also derived, which are consistent with those obtained by numerical simulation we conducted.

Furthermore, we compare the trapping efficiency and diffusion efficiency between any two nodes of NFSFT by using the MTT and the MDT as the measures of trapping efficiency and diffusion efficiency respectively, and find the best (or worst) trapping sites and the best (or worst) diffusing sites. Our results show that the two hubs of NFSFT is not only the best trapping site, but also the worst diffusing site, and that the nodes which are the farthest nodes from the two hubs are not only the worst trapping sites but also the best diffusion sites. Finally, we find that the ratio between the maximum and minimum of MTT grows logarithmically with network order, whereas the ratio between the maximum and minimum of MDT is almost equal to 1, by comparing the maximum of MTT and MDT with their minimum. Thus the trap's position has great effect on the trapping efficiency, but the position of starting node almost has no effect on diffusion efficiency.

2. The network model and some notions

The recursive non-fractal scale-free trees (NFSFT) we considered can be constructed iteratively [38]. For convenience, we call the times of iterations as the generation of the NFSFT and denote by $G(t)$ the NFSFT of generation t. For $t = 0$, $G(0)$ is an edge connecting two nodes. For $t > 0$, $G(t)$ is obtained from $G(t-1)$: for each of the existing edges in $G(t-1)$, we introduce $2m$ $(m > 0)$ new nodes, half of them are connected to one end of the edge, and half of them are linked to the other end. That is, $G(t)$ is obtained from $G(t-1)$ via replacing every edge in $G(t-1)$ by the cluster on the right-hand side of the arrow in Fig. 1. The construction of NFSFT of generation 3 for the particular case of $m = 1$ is shown in Fig. 2.