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Spontaneous generation of a crystalline ground state in a higher derivative theory

Subir Ghosh *

Physics and Applied Mathematics Unit, Indian Statistical Institute, 203 B. T. Road, Kolkata 700108, India

HIGHLIGHTS

- Higher derivative theory.
- Spontaneous Symmetry Breaking in momentum space.
- Possible application in Horava gravity.

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ABSTRACT

The possibility of Spontaneous Symmetry Breaking in momentum space in a generic Lifshitz scalar model – a non-relativistic scalar field theory with higher spatial derivative terms – has been studied. We show that the minimum energy state, the ground state, has a lattice structure, where the translation invariance of the continuum theory is reduced to a discrete translation symmetry. The scale of translation symmetry breaking (or induced lattice spacing) is proportional to the inverse of the momentum of the condensate particle. The crystalline ground state is stable under excitations below a certain critical velocity. The small fluctuations above the ground state can have a phonon like dispersion under suitable choice of parameters.

At the beginning we have discussed the effects of next to nearest neighbor interaction terms in a model of linear triatomic molecule depicted by a linear system of three particles of same mass connected by identical springs. This model is relevant since in the continuum limit the next to nearest neighbor interaction terms generate higher (spatial) derivative wave equation, the main topic of this paper.

1. Introduction

Generating a continuum field theory as a limiting case of a discrete set of coupled dynamical systems in the limit when the spacing between the systems goes to zero is common. A text book example is the (nearest neighbor) coupled chain of harmonic oscillators that reduces to the continuum elastic wave theory of sound in the limit when the oscillator spacing goes to zero. (See for example [1].)

On the other hand, quite surprisingly, example of the complimentary phenomenon where a continuum field theory can pass on to a theory with discrete degrees of freedom, (or more weakly where the continuous nature of the ground state is replaced by a discrete structure), is comparatively rare. In the present paper we argue that indeed this process can also be quite common and can occur in any generic higher derivative theory. We provide an explicit model, *a higher derivative*

* Tel.: +91 33 25753362; fax: +91 33 25773026.
E-mail addresses: sghosh@isical.ac.in, subir_ghosh2@rediffmail.com.

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Lifshitz theory, that yields a discrete ground state with a lattice structure. We also discuss the stability criteria of such a ground state and properties of small excitations above the ground state.

The necessity of higher derivative terms has been emphasized in a series of papers by Polonyi and collaborators [2–5], in the context of relativistic models where an instability in momentum space induces Spontaneous Symmetry Breaking, yielding a discrete latticised ground state. Our present work is very close in spirit to Ref. [2] where the authors have revealed the rich phase structure of a higher derivative ϕ^4-theory, based on an inhomogeneous (periodic) ground state, along with its one-loop renormalization. Further, the model is renormalized to all orders in a perturbative framework and is shown to be unitary in Ref. [3]. It is well known that a relativistic higher derivative field theory is plagued with ghost excitation problem [6]. This issue is discussed in detail in Ref. [4] in the context of scalar QED with higher derivative terms where the gauge invariance plays a crucial role. This novel phenomenon is extended to non-Abelian gauge theory with higher derivatives in Ref. [5]. The interesting aspect in these works is that in situations where the higher derivative term in the kinetic part dominates, it can lead to a Spontaneous Symmetry Breaking (SSB) in momentum space, resulting in a dynamical breaking of Lorentz invariance. The ground state is given by a condensate where the particles carry a non-zero momentum yielding an inhomogeneous spacetime dependent ground state that breaks translation invariance. The (length) scale of inhomogeneity is inversely proportional to the particle momentum in the condensate.

As we have mentioned above, our work closely resembles [2] with the important distinction that we have considered a theory where Lorentz invariance is explicitly broken by the higher derivative terms. Hence we avoid the ghost problem (of higher derivative relativistic theories) altogether and SSB of Lorentz invariance is not an issue. No new excitations, Goldstone modes or otherwise, are generated. In our model, a non-relativistic theory is considered where the higher *time* derivatives, responsible for the ghost, are dropped and only spatial higher derivative terms are kept. In effective theories higher derivative terms are naturally generated when one removes (or integrates out) some degrees of freedom. Such a theory, known as Lifshitz theory [7], is quite familiar in Condensed Matter Physics. But our main interest lies in applying this form of momentum space SSB in High Energy Physics, especially in Quantum Gravity models. In recent years Horava has proposed a conventional field theory for gravity [8] where spatial higher derivative terms make the theory UV complete, thereby serving as a candidate of a Quantum Gravity model. It is important to stress that a restricted form of Horava gravity model is essentially same as the model studied in the present paper, (as shown by our earlier work [9]). Hence the results and conclusions reached here should be relevant not only in contexts of Condensed Matter Physics but also in Horava theory of gravity as well.

In the present paper we plan to study an interesting aspect of higher derivative generic Lifshitz models which so far have not been investigated. The higher derivative terms can induce an instability leading to phase transition. Furthermore, the ground state (the lowest energy state) turns out to be crystalline with the lattice spacing dependent on the higher derivative coupling constant. Hence the continuous translation symmetry of the system is broken by the ground state that enjoys a discrete translation symmetry only.

The above framework reminds us of the celebrated Landau theory [10] of liquid solid phase transition where Spontaneous Symmetry Breaking (SSB) leads to a state with less symmetry due to the structures present, the solid, from a more uniform and hence symmetric state, the liquid. In a series of works, Alexander and Mctague [11] have shown in an essentially model independent way how crystalline lattice structure emerges from liquid. Rabinovici et al., have utilized these ideas in the context of String Theory compactification [12]. In the present case, a similar thing happens: SSB generates a less symmetric crystalline vacuum condensate. The higher derivative terms, a signature of the Lifshitz scalar model, are essential in inducing the inhomogeneous condensate. Very recently somewhat similar ideas have been suggested by Wilczek and by Shapere and Wilczek in Ref. [13].

The paper is organized as follows: Section 2 deals with a mechanical analogue toy model. In Section 3 we briefly recapitulate SSB in a conventional scalar theory. In Section 4 we present our main results, the effects of SSB in momentum space in a generic Lifshitz model. The paper ends with conclusions and future outlook in Section 5.

2. Mechanical analogue model for higher derivative wave equation

Below we try to analyze the effect of higher derivative term in a simple analogue mechanical model. Although this toy model is not fully satisfactory still it indicates that the ground state of the system can be affected by such higher derivative (analogue) terms in a way that is of interest for the present work. We exploit the fact that higher derivative terms can be simulated in a wave equation by *next* to nearest neighbor interactions in the continuum limit of a linear chain of mass points connected by springs.

It is straightforward to generate the (elastic) wave equation from a chain of mass points connected by oscillators with nearest neighbor interactions, in the limit of the equilibrium separation between the mass points, a, going to zero (see for example [1]). With η_i denoting displacement of the ith mass point from its equilibrium position and $\frac{d\eta}{dt} = \dot{\eta}$, $\frac{d\eta}{dx} = \eta'$ the Lagrangian of the chain is

$$L = \frac{1}{2}\sum_i [m\dot{\eta}_i^2 - \kappa(\eta_{i+1} - \eta_i)^2]$$
$$= \frac{1}{2}\sum_i a\left[\frac{m}{a}\dot{\eta}_i^2 - (\kappa a)\left(\frac{\eta_{i+1} - \eta_i}{a}\right)^2\right]. \quad (1)$$