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Minireview

Explosive percolation: Unusual transitions of a simple model

N. Bastas [a], P. Giazitzidis [a], M. Maragakis [a,b], K. Kosmidis [a,c,d,*]

[a] *Physics Department, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece*
[b] *Department of Economics, University of Macedonia, Thessaloniki, Greece*
[c] *Computing & Communications Center, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece*
[d] *School of Engineering and Science, Jacobs University Bremen, Campus Ring 1, 28759 Bremen, Germany*

HIGHLIGHTS

- Review of the field of explosive percolation.
- Collection of all the theoretical and numerical findings listed in tables.
- Review of several explosive percolation related models.
- Guidelines for future research.

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ABSTRACT

In this paper we review the recent advances in explosive percolation, a very sharp phase transition first observed by Achlioptas et al. (2009). There a simple model was proposed, which changed slightly the classical percolation process so that the emergence of the spanning cluster is delayed. This slight modification turns out to have a great impact on the percolation phase transition. The resulting transition is so sharp that it was termed explosive, and it was at first considered to be discontinuous. This surprising fact stimulated considerable interest in "Achlioptas processes". Later work, however, showed that the transition is continuous (at least for Achlioptas processes on Erdös networks), but with very unusual finite size scaling. We present a review of the field, indicate open "problems" and propose directions for future research.

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* Corresponding author at: Physics Department, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece.
E-mail address: kosmask@auth.gr (K. Kosmidis).

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1. Introduction

Percolation is a very important topic in the Statistical Physics of phase transitions [1,2]. Although its first appearance in the scientific literature dates back to the year 1940 and the work of Flory [2] it remains a challenging and active field of research even today. A brief search in a scientific search-engine like "Scopus" reveals that last year (2012) more than 1800 scientific papers containing the word "Percolation" in their abstracts have been published in peer-reviewed journals. This is a clear indication of the activity of the field and its interest for the scientific community. Percolation represents a paradigmatic model of a geometric phase transition. In the classical site percolation model, the sites of a square lattice are randomly occupied with particles with probability p, or remain empty with probability $1-p$. Neighboring occupied sites are considered to belong to the same cluster. Percolation theory simply deals with the number and properties of these clusters. When the occupation probability p is small, the occupied sites are either isolated or form very small clusters. On the other hand, for large p there are a lot of occupied sites that have formed one large cluster. It is in fact possible to find several paths of occupied sites which a walker can use to move from one side of the lattice to the other. In this latter case, it is said that a giant component of connected sites exists in the lattice. This "infinite cluster" as it is called does not appear in a gradual "linear" way with increasing p. It appears above a critical occupation probability p_c. Below p_c there are only small clusters and even if we increase the lattice size considerably, these clusters remain small, i.e. the size of the largest cluster does not depend on the system size. Above p_c, suddenly, small clusters join together to form a single large cluster whose size scales with the system size. Hence, the term giant component or infinite cluster which is very common in the literature [1].

Thus, the phase transition related to Percolation is a geometric one, i.e. the appearance of an "infinite" connected cluster. In Physics, phase transitions are usually thermally induced, meaning that a property appears above or below a characteristic temperature. During a phase transition of a given medium certain properties of the medium change, often discontinuously, as a result of some external condition, such as temperature, pressure, and others. For example, a liquid may become gas when heated up to the boiling point, resulting in an abrupt change in volume. Phase transitions can be described by determining the behavior of an "order parameter". The order parameter is normally a quantity which is zero in one phase (usually above the critical point), and non-zero in the other. For a liquid–gas transition the difference of the densities of the gas ρ_{gas} and the liquid phase ρ_{liq}, $\rho = |\rho_{\text{liq}} - \rho_{\text{gas}}|$ is an appropriate order parameter.

Phase transitions are marked by a singularity of the free energy or one of its derivatives at the transition point. First-order phase transitions exhibit a discontinuity in the first derivative of the free energy with respect to some thermodynamic variable. In this case the order parameter is discontinuous, as for an example in a solid–liquid phase transition. First-order phase transitions are those that involve a latent heat. During such a transition, a system either absorbs or releases a fixed (and typically large) amount of energy. During this process, the temperature of the system will stay constant as heat is added: the system is in a "mixed-phase regime" in which some parts of the system have completed the transition and others have not. Familiar examples are the melting of ice or the boiling of water (the water does not instantly turn into vapor, but forms a turbulent mixture of liquid water and vapor bubbles).

When the change of the order parameter is not discontinuous, we usually talk about second-order transitions. Second-order phase transitions are also called continuous phase transitions. They are characterized by a divergent susceptibility, an infinite correlation length, and a power-law decay of correlations near criticality. Examples of second-order phase transitions are the ferromagnetic transition, the superconducting transition and the superfluid transition.

In percolation, p plays the same role as the temperature in thermal phase transitions, i.e. that of the control parameter, while the order parameter is the probability P_∞ that a site belongs to the infinite cluster. Classical percolation exhibits all the characteristics of a continuous phase transition. For $p > p_c$, P_∞ increases with p by a power law

$$P_\infty \sim (p - p_c)^\beta. \tag{1}$$

Other important quantities are the correlation length ξ which is defined as the mean distance between two sites on the same finite cluster and the mean number of sites S of a finite cluster. When p approaches p_c, ξ increases as

$$\xi \sim (p - p_c)^{-\nu}. \tag{2}$$

The mean number of sites S of a finite cluster also diverges at p_c

$$S \sim (p - p_c)^{-\gamma}. \tag{3}$$

The critical exponents β, ν and γ describe the critical behavior associated with the percolation transition and are universal. Although, the percolation threshold changes for even slight modifications of the model (for example, site and bond p_c's are different) the critical exponents are very robust in changing the percolation model details [3]. They do not depend on the structure of the lattice (e.g., square or triangular) or on the type of percolation (site, bond or even continuum) [2].

Thus, there was a big surprise when recently, Achlioptas et al. [4], claimed that a rather simple modification of the original percolation process leads to a new seemingly first-order percolation transition. This transition was named "Explosive percolation" (EP) due to the abrupt character of the transition. However, percolation models which are characterized by first-order transition were reported more than 20 years ago. For example, in Ref. [5], bootstrap percolation on hypercubic lattices was shown to exhibit discontinuous transition for certain bounds. What is new with this process is that it depends on the site (or bond) occupation history, thus falling into the category of non-equilibrium processes (unlike other percolation models). In this paper, we review the recent developments on the investigation of this "explosive" transition presenting the most important results concerning the nature of the transition as well as the several variants of the original model.