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Multifractal and singularity analysis of highway volume data

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HIGHLIGHTS

- The traffic data on work days and rest days have a similar fluctuation.
- Hölder exponents among a day, a month, three months and a holiday's data.
- The singularity exponent method is more precise than the Hölder exponent method.

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ABSTRACT

Recent work has shown that the mathematics of multifractal can be used to provide a quantitative signature in many fields. In this paper, we investigate the traffic time series for volume data observed on Guangshen highway. Firstly, we find there exists a multifractal behavior in the traffic data, and the data on both work days and rest days have similar multifractality. Then, we study the singularity of these data. A singularity exponent method based on multifractal theory is proposed to extract or classify singular data, which is more precise and clear than the Hölder exponent method.

1. Introduction

A fractal is a mathematical set that has one fractal dimension which usually exceeds its topological dimension and may fall between the integers. Fractals are typically self-similar patterns, which means "the same from near as from far". The features of fractals are also characteristics of many naturally occurring objects and phenomena, such as images, structures and sounds. However, most natural fractals do not tend to have merely one scaling process, for example, turbulence [1,2], diffusion limited aggregation [3], signal processing [4], central place system [5], sheep livestock price [6], filters [7], urban morphology [8,9], oil markets [10], soil pore system [11] and so on. Such fractals are called multifractal, which are characterized by a sequence of fractal dimensions rather than just one fractal dimension. They also have an associated multi-fractal spectrum. In the past few years, multifractal analysis has been widely used and studied as a mathematical tool in many applications [1,3,4,12,13].

Multifractal is not only limited to describe geometric patterns, but can also describe process in time series. So attributes of the theory should be cast in transportation system whose current state and future evolution depend greatly on many properties of interactions between physical and human elements. This can make a great contribution to transportation planning and forecasting [14–18]. Vojak et al. studied the structure of road traffic and proposed an approach for short-term prediction for traffic system [18]. Véhel and Sikdar proposed a multiplicative multifractal model for TCP traffic [4]. Then, Shang et al. investigated the presence of multifractal behaviors in the traffic time series of one day by statistical moment scaling method and Hölder exponent method [19,20].

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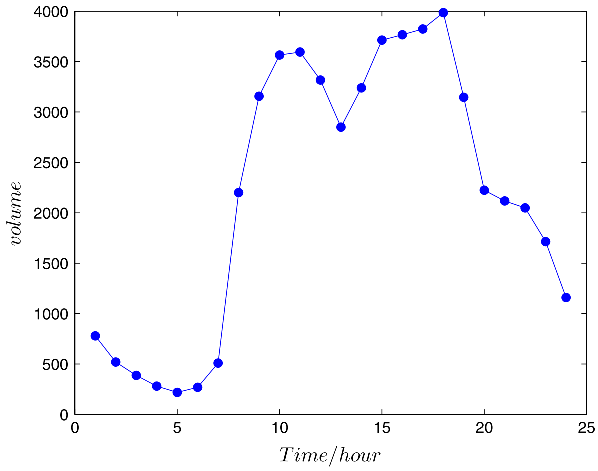

Fig. 1. Plot of the volume data from Guangshen highway for 24 h averaged on three months.

This paper continues the examination of the multifractal methodology as a tool for the transportation system. Compared to previous works, we made some improvements. Firstly, we aim to investigate the existence of multifractal in traffic data observed on Guangshen highway, and find that the data on both work days and rest days have similar multifractality. Secondly, we find that four curves of Hölder exponents for 24 h averaged on a normal day, a month, three months and one holiday, respectively, are similar to some extent. Furthermore, we propose an original singularity exponent method to calculate the singularity exponents for each time point, which is more precise than the Hölder exponent method [20]. Based on our method, it can be easy to find the irregular or normal hours in one day.

The organization of this paper is as follows. In Sections 2 and 3, we introduce the traffic data and investigate the multifractality of the volume data on both work days and rest days. Then, in Section 4, we analyze the singularity of our data averaged on a normal day, a month, three months and one holiday. Besides, the theory and method for the calculation of singularity exponents are proposed. And in Section 5, we make conclusions.

2. Traffic data

We use the data observed on Guangshen highway over a period of three months, including 58 work days and 32 rest days, from 1 February in 2008 to 30 April in 2008. The raw data we obtained are the number of cars passed the Meiling checkpoint in every five minutes. For ease of viewing the traffic situation, we aggregate these data into one-hour data for average volume (see Fig. 1) (i.e. the average number of cars passed in an hour). However, in order to get the more accurate results, we still use the five-minute data in the following analysis.

3. Multifractal analysis of Guangshen highway volume data

In this section, we will use the statistical moment scaling method [20] to investigate the existence of the multifractal in traffic data observed on Guangshen highway. This statistical moment scaling method is the most widely used one in time series among the existing multifractal tools. Unlike Ref. [20], we study two multifractal spectra of our volume data both on work days and rest days, where rest days include weekdays and holidays.

We divide the time series $\{y_1, y_2, \ldots, y_n\}$ into non-overlapping intervals with a certain time resolution. Define k as a scale ratio, representing the ratio of the maximum scale of the field to this interval. According to different values of k, we can calculate the average intensity in each interval i, and denote it by $\mu(k, i)$

$$\mu(k,i) = \frac{N(k,i)}{S}, \tag{1}$$

where S is the sum of all values in the time series and $N(k, i)$ is the sum of the values belonging to the interval i.

Since the Hausdorff multifractal spectrum of $\mu(k, i)$ describes the 'size' of the subset of points having the same scaling behavior, it can be used to quantize the singular structure of the measure $\mu(k, i)$. Hausdorff multifractal spectrum of the measure $\mu(k, i)$ is defined by

$$f_H(\alpha) = \dim_H(E_\alpha),$$

where

$$E_\alpha = \left\{ i \in (1,n) : \lim_{k\to\infty} \frac{\log \mu(k,i)}{-\log k} = \alpha \right\},$$

the definition is most useful in purely mathematical settings.