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Hybrid evolving clique-networks and their communicability

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HIGHLIGHTS

- We propose a hybrid clique network model to mimic the behaviors of real-world hierarchical networks.
- An *inhomogeneity parameter* is introduced to tune the fraction of the inhomogeneous parts of the networks.
- The numerical investigations on the properties and communicability of this network are presented.

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ABSTRACT

Aiming to understand real-world hierarchical networks whose degree distributions are neither power law nor exponential, we construct a hybrid clique network that includes both homogeneous and inhomogeneous parts, and introduce an inhomogeneity parameter to tune the ratio between the homogeneous part and the inhomogeneous one. We perform Monte-Carlo simulations to study various properties of such a network, including the degree distribution, the average shortest-path-length, the clustering coefficient, the clustering spectrum, and the communicability.

1. Introduction

The studies of complex networks have thrived over the past decades [1–4]. Various scholars have proposed many models to mimic the features of complex networks, including the Erdös–Rényi (ER) random-network model [5], the small-world-network model [6], and the scale-free-network model [7]. In most of these models, the networks are constructed from individual nodes [8–11]. In 2005, Derenyi et al. introduced the concept of "clique percolation" [12], and Palla et al. used a method based on this concept to explore overlapping communities [13]. A clique of size a is a completely interconnected unit that includes a nodes and $a(a-1)/2$ edges. Different from scale-free networks, networks constructed from cliques possess hierarchical structures, and are able to offer deep insights into real-world networks whose central organizing principle is hierarchy [14–20].

Takemoto et al. introduced a model studying evolving hierarchical networks constructed by reorganizing cliques according to the preferential attachment rule [21], and later several similar models are proposed. These models are applicable to inhomogeneous networks, which have power-law degree distributions [22,23]. However, many real-world networks are homogeneous. The homogeneity of a network means that almost all nodes of the network are topologically equivalent to one another, like those in regular lattices or in random graphs; and a homogeneous network usually has an exponential (or Poisson) degree distribution. To mimic the behaviors of these (homogeneous and hierarchical) networks, we constructed in

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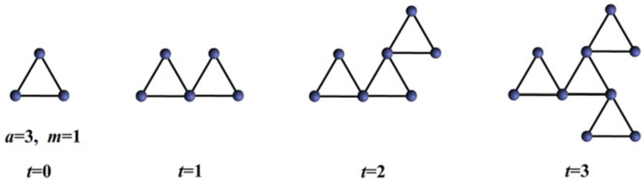

Fig. 1. Schematic illustration of the evolutionary procedure of our model network for $a = 3$ and $m = 1$. Each clique is attached through common nodes with others, and no extra edge is added during the attaching process. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Ref. [24] a network by attaching cliques randomly, and the networks formed this way possess the desired properties, *viz.*, the networks are homogeneous and hierarchical.

In this paper, we study clique networks including both inhomogeneous and homogeneous parts. We introduce an *inhomogeneity parameter p* which specifies the fraction of the homogeneous (or inhomogeneous) part in the whole network. By tuning the value of p, we are then able to change the network properties, in particular, the degree distribution. When p decreases from 1 to 0, the degree distribution changes from a power-law distribution to an exponential one because a completely inhomogeneous network is changed to a homogeneous one. For $0<p<1$, our model can mimic the behaviors of real-world networks whose degree distributions are neither power law nor exponential. This paper is organized as follows. In Section 2, we describe how to construct our model network; in Section 3, we present the numerical results of the network properties; the study on the network communicability is reported in Section 4; a brief conclusion is presented in the last section.

2. Model

We consider a network composed of cliques of size a ($a > 2$). The cliques are linked to one another through common nodes and there are no extra edges added during the process of attaching (see Fig. 1). At every time step, a new clique is attached to the network at m ($m < a$) nodes. The selection rule of the attaching is specified as follows: (i) at a given probability p, the new clique selects the m attachment nodes according to the preferential attachment (PA) rule, *viz.*, the probability of node i being attached by the new clique, Π_i, is given by

$$\prod_i = \frac{k_i}{\sum_j k_j} \tag{1}$$

where k_i is the degree of node i and the sum runs over all nodes of the network (excluding the new clique); (ii) at probability $1 - p$, the new clique selects its attachment nodes in a completely random way, *viz.*, $\Pi_i = 1/N(t)$, where $N(t)$ represents the total node number of the network at time step t (excluding the new clique). Obviously, for $p = 1$, our model is the same as the Takemoto–Oosawa model and becomes the scale-free model for $a = 2$ and $m = 1$; when $p = 0$, this model is the same as the one reported in Ref. [24] and becomes the complete random network model for $a = 2$ and $m = 1$. For a network with large N, the parameter p specifies the fraction of the network that is inhomogeneous, and we thus call it *inhomogeneity parameter*.

3. Network properties

We perform Monte Carlo simulations to study the properties of our model networks. We investigate (i) the degree distributions, (ii) the average shortest-path-lengths, (iii) the clustering coefficients, and (iv) the clustering spectra of networks having same node number and average degree but different inhomogeneity parameters. We focus on networks constructed from cliques with size $a = 5$ and the number of attachment nodes $m = 1$, 2 or 3. During the simulations, we tune the number of time steps to obtain networks with desired node numbers (N) and average degrees ($\langle k \rangle$).

3.1. Degree distribution

The most important topological property of a complex network is its degree distribution $P(k)$ or cumulative degree distribution $CP(k)$. Considering that $CP(k)$ can reduce the noise in the tail of distribution curves, we present in Fig. 2 the numerical values of $CP(k)$ for $p = 0$, 0.3, 0.5, 0.7, and 1, where we have set $N = 5000$, $a = 5$, $m = 2$, and $\langle k \rangle = 6.66$. The inset of Fig. 2 shows that the relation between $CP(k)$ and k is represented by a straight line in a semi-log plot for $p = 0$, implying that $CP(k)$ decays exponentially with k, which agrees with the results of a homogeneous network. For $p = 1$, the relation between $CP(k)$ and k in a log–log plot is a straight line whose slope is approximately -1.48, indicating that $CP(k) \propto k^{-\gamma}$, with $\gamma = 1.48$, and that the degree distribution $P(k) \propto k^{-\gamma-1}$. The exponent of the power-law degree distribution is $2 < \gamma + 1 < 3$, which agrees with the values reported elsewhere [1,3,4]. Furthermore, we can see from Fig. 2 that when p decreases from 1 to 0 the curves of $CP(k)$ vs. k in the log–log plots bend more and more (away from a