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Sandpile on directed small-world networks

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Abstract

We numerically investigate the avalanche dynamics of the Bak–Tang–Wiesenfeld sandpile model on directed small-world networks. We find that the avalanche size and duration distribution follow a power law for all rewiring probabilities p. Specially, we find that, approaching the thermodynamic limit ($L \to \infty$), the values of critical exponents do not depend on p and are consistent with the mean-field solution in Euclidean space for any $p > 0$. In addition, we measure the dynamic exponent in the relation between avalanche size and avalanche duration and find that the values of the dynamic exponents are also consistent with the mean-field values for any $p > 0$.

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1. Introduction

Small-world (SW) networks, recently introduced by Watts and Strogatz [1], have attracted a lot of interest [2,3]. The model results from rewiring a fraction of links on a d-dimensional regular lattice with a probability p to other vertices. As a result of the random rewiring, one can closely monitor the transition between order ($p = 0$) and randomness ($p = 1$) by varying p. SW networks with symmetric links are called undirected networks; they are directed if instead the relation links have a direction. Many researches show that directed links play an important role in trying to model real systems [4,5]. It would be very interesting to study the effect of directed links on some classical model evolving on the SW network. Recently, Sanchez et al. have studied the behavior of a simple spin-like model evolving on a directed SW network [6]. They found that the existence of directed links completely changes the behavior of the system from the mean-field behavior (for undirected networks) to a highly nontrivial and rich phase diagram including first- and second-order phase transitions out of equilibrium.

Avalanching behavior as well as scale invariance has also been experimentally observed in a variety of situations in nature [7–10]. In order to describe the appearance of the scale invariance, Bak, Tang, and

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Wiesenfeld (BTW) introduced the concept of self-organized criticality (SOC) [11]. The term refers that an externally driven physical system with many degrees of freedom can be spontaneously organized into a dynamical critical state. In general, the systems are driven at a very slow time scale and relax with bursts of activity, avalanches, on a very fast time scale. At the critical state, the SOC systems exhibit avalanche dynamics with long-range spatial and temporal correlations, and the avalanche sizes follow a power-law distribution, i.e., $p(s) \sim s^{-\tau}$ with the critical exponent τ.

The BTW sandpile [11] model is a prototypical theoretical model of SOC. In the past, more than 10 years, many studies of the sandpile model have been carried out on undirected and directed Euclidean space. The results show that the critical behaviors of the avalanche on directed Euclidean space are completely different from those on undirected Euclidean space [12–15]. Recently, the study of it on complex networks has attracted a lot of interest. Goh et al. [16] studied the BTW sandpile model on the undirected scale-free networks, where the degree distribution follows a power law ($p_d(k) \sim k^{-\gamma}$). They found the critical exponent $\tau = 1.5$ for $\gamma < 3$, consistent with the mean-field solution in Euclidean space, and $\tau < 1.5$ for $2 < \gamma < 3$. de Arcangelis and Hermann [17] studied the BTW sandpile model on the undirected SW networks and found for p close to 1 the value of critical exponent convert to the mean-field value $\tau = 1.5$. While some studies have been performed on undirected complex networks, the study of BTW sandpile model on directed complex networks has not been carried out yet. What is the effect of directed links on BTW sandpile model evolving on complex networks?

In this paper, we study the dynamics of BTW sandpile model on a directed SW network. In case only undirected links are used, our model becomes identical to the BTW sandpile model on an undirected SW network studied [17]. By means of extensive numerical simulations, we find that, approaching the thermodynamic limit ($L \to \infty$), the values of critical exponents do not depend on p and are consistent with the mean-field solution in Euclidean space for any $p > 0$, which is completely different from the case of undirected SW networks.

2. Model

We use the directed SW networks introduced by Sanchez et al. [6]. We start from a two-dimensional square lattice consisting of sites linked to their four nearest neighbors by both outgoing and incoming links as shown in Fig. 1. And then we reconnect nearest-neighbor outgoing links to a different site chosen at random with probability p. For every outgoing link, we repeat this process. Finally, we obtain a network with density p of SW directed links. In this network, each site has exactly four outgoing links and a varying number of incoming links. It is noted that the sites on the boundary have directed links to the 'outside'; however, these links are not rewired.

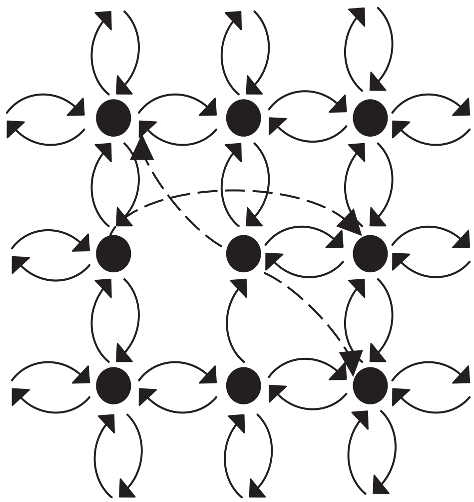

Fig. 1. Sketch of a directed small-world network constructed from a two-dimensional square regular lattice; the arrows indicate the direction of the corresponding link, and dotted lines represent rewired links.