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Characterizing abrupt changes in the stock prices using a wavelet decomposition method

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Abstract

Abrupt changes in the stock prices, either upwards or downwards, are usually preceded by an oscillatory behavior with frequencies that tend to increase as the moment of transition becomes closer. The wavelet decomposition methods may be useful for analysis of this oscillations with varying frequencies, because they provide simultaneous information on the frequency (scale) and localization in time (translation). However, in order to use the wavelet decomposition, certain requirements have to be satisfied, so that the linear and cyclic trends are eliminated by standard least squares techniques. The coefficients obtained by the wavelet decomposition can be represented in a graphical form. A threshold can then be established to characterize the likelihood of a short-time abrupt change in the stock prices. Actual data from the São Paulo Stock Exchange (Bolsa de Valores de São Paulo) were used in this work to illustrate the proposed method.

Keywords: Wavelets; Stocks; Prices; Abrupt changes; Localization

1. Introduction

Stock Markets have experienced rapid growth during recent years in many countries and efforts have been directed into studying their dynamic behavior. One line of research attempts to adapt physical and mathematical modeling techniques with a view on characterizing trends in the stock market prices (e.g. Refs. [1,2,3–11], among others).

A successful characterization of the dynamics of the stock prices, particularly of sudden large drops, can have a profound impact on risk management. Some of the many quantitative concepts and tools that are used to assist the decision making process include: moving average modeling, intra-day maximum and minimum price estimates, volume analysis, linear regression, correlation, support and resistance, moment sensor, cycle study, relative force index and others.

One very interesting approach considers that the stock prices have a dynamic behavior that resembles, very closely, some physical phenomena [3–5,9,10]. In particular, Johansen [3], showed that the behavior of the

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prices in the stock market, just prior to abrupt changes, present a profile very similar to those observed in the earthquake phenomenon. Using a different approach, Chen [12,13] showed, with the aid of mathematical models of two competing species adapted for the case of financial assets, that it may be possible to observe the occurrence of bifurcation and chaos.

In these studies, an interesting feature that is frequently mentioned is the onset of small amplitude and high-frequency fluctuations in the buy–sell actions. Johansen [3] proposed a mathematical model involving an almost sinusoidal oscillation with amplitudes that tend to become smaller and smaller, while the frequency that tends to increase, as an abrupt change (crash) becomes imminent.

Here, after pre-processing the raw data by adapting the method used in Ref. [13], the wavelet decomposition technique is applied. Wavelets are also used in Ref. [8] to study the reaction of markets to crashes. The resulting coefficients are then plotted on a graph. Now, it is possible to visualize graphically the dynamic behavior of data modeled as in Ref. [3], so that some simple rules based on thresholds can be heuristically established to characterize the eventual occurrence of short-time abrupt changes in the stock prices.

As examples, data from the São Paulo Stock Exchange (BOVESPA) were used, namely, a price index (IBOVESPA) and stock prices of major companies (Usiminas—USIM5, Gerdau—GGBR4 and Embratel—EBTP4).

2. A mathematical model of market crash

The mathematical model for market crashes proposed by Johansen [3] is closely related to sysmic data registered during earthquakes. Basically, after a long period of mild fluctuations, an onset of chirp type signal is observed just prior to the earthquake, as described by equations of form

$$y(t) = A + B \times (t_c - t)^{\alpha} + \frac{B \times C \times (t_c - t)^{\alpha}}{(t_c - t)} \times \cos(\omega \times \ln(t_c - t) + \phi), \tag{1}$$

where y is the price index, A–C are constant parameters and t_c represents the crash time. For instance, if the BOVESPA index from August 9, 1999 up to June 2, 2000 is fitted to Eq. (1) by a standard least squares procedure, the values of the model parameters turn out to be $A = 2000$; $B = -6$; $C = 19$; $\alpha = 1.33$; $\omega = 20$; $\phi = 5.9$. The abrupt change would then be expected to occur around day 140. In fact, IBOVESPA presented a draw down of about 33% starting at day 142. This suggests that small fluctuations in the market prices contain important information on imminent occurrence of abrupt changes. Intuitively, the oscillations with small amplitudes and high frequency next to the crash time suggest that the traders are buying and selling intensively with reduced price differences.

3. The wavelet analysis

In this work, the wavelet decomposition technique is used instead of the explicit mathematical model expressed by Eq. (1). The idea is that wavelet analysis provide simultaneous information on the frequency (scale) and localization in time (translation), so that onset of small amplitude chirp-type signals can be detected. In general terms, long time intervals (greater scales) are related to low-frequency information, while shorter intervals to higher frequencies. Therefore, if the coefficients of the wavelet decomposition are represented in a scales $\times$ time chart, one can detect the tendency of higher frequency to become more prominent, thus indicating the proximity of an abrupt change in the stock prices.

In a rough manner, a wavelet is a waveform of effectively limited duration that has the time-average value equal to zero. In can be shown that under some assumptions (see for example Refs. [14–16]), a family of orthonormal wavelets ψ can be constructed by scaling and translating a single function φ, called mother wavelet. The idea is to use ψ as a basis to represent a function in an expanded form. The procedure is analogous to that of expressing a periodic function as a sum of sines and cosines with appropriate weights or coefficients.