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Evolutionary snowdrift game with an additional strategy in fully connected networks and regular lattices

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Abstract

The effects of an additional strategy or character in the evolutionary snowdrift game (SG) are studied in a well-mixed population or fully connected network and in a square lattice. The SG, which is a possible alternative to the prisoner's dilemma game in studying cooperative phenomena in competing populations, consists of two types of strategies, C (cooperators) and D (defectors). The additional L-strategy amounts to a strongly persuasive character that a fixed payoff is given to each player when a L-player is involved, regardless of the character of the opponent. In a fully connected network, it is found that either C lives with D or the L-players take over the whole population. In a square lattice, three possible situations are found: a uniform C-population, C lives with D, and the coexistence of all three characters. The presence of L-players is found to enhance cooperation in a square lattice by enhancing the payoff of cooperators. The results are discussed in terms of the effects in restricting a player to compete only with his nearest neighbors in a square lattice, as opposed to competing with all players in a fully connected network.

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1. Introduction

The evolutionary prisoner's dilemma game (PDG) [1–3] and the snowdrift game (SG) [4] have become standard paradigms for studying the possible emergence of cooperative phenomena in a competitive setting. Physicists find such emergent phenomena fascinating, as similar cooperative effects are also found in interacting systems in physics that can be described by some minimal models, e.g. models of interacting spin systems. These games are also essential in the understanding of coexistence of (and competition between) egoistic and altruistic behavior that appear in many complex systems in biology, sociology and economics. The basic PDG [5,6] consists of two players deciding simultaneously whether to cooperate (C) or to defect (D).

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If one plays C and the other plays D, the cooperator pays a cost of $S = -c$ while the defector receives the highest payoff $T = b$ ($b > c > 0$). If both play C, each player receives a payoff of $R = b - c > 0$. If both play D, the payoff is $P = 0$. Thus, the PDG is characterized by the ordering of the four payoffs $T > R > P > S$, with $2R > T + S$. In a single round of the game, defection is a better action in a fully connected (well-mixed) population, regardless of the opponents' decisions. Modifications on the basic PDG are, therefore, proposed in order to induce cooperations and to explain the wide-spread cooperative behavior observed in the real world. These modifications include, for example, the iterated PDG [1,2], spatially extended PDG [7–10] and games with a third strategy [11–14].

The SG, which is equivalent to the hawk-dove or chicken game [4,15], is a model that somewhat favors cooperation. It is best introduced using the following scenario [16]. Consider two drivers hurrying home in opposite directions on a road blocked by a snowdrift. Each driver has two possible actions—to shovel the snowdrift (cooperate (C)) or not to do anything (not-to-cooperate or "defect" (D), following the PDG notations). If they cooperate, they could be back home earlier and each will get a reward of b'. Shovelling is a laborious job with a total cost of c'. Thus, each driver gets a net reward of $R = b' - c'/2$. If both drivers take action D, they both get stuck, and each gets a reward of $P = 0$. If only one driver takes action C and shovels the snowdrift, then both drivers can also go home. The driver taking action D (not to shovel) gets home without doing anything and hence gets a payoff $T = b'$, while the driver taking action C gets a "sucker" payoff of $S = b' - c'$. The SG refers to the case when $b' > c' > 0$, leading to the ranking of the payoffs $T > R > S > P$. This ordering of the payoffs *defines* the SG. Therefore, both the PDG and SG are defined by a payoff matrix of the form

$$\begin{array}{cc} & \begin{matrix} C & D \end{matrix} \\ \begin{matrix} C \\ D \end{matrix} & \begin{pmatrix} R & S \\ T & P \end{pmatrix}, \end{array} \quad (1)$$

and they differ only in the ordering of P and S. It is this difference that makes cooperators persist more easily in the SG than in the PDG. In a well-mixed population, cooperators and defectors coexist. Due to the difficulty in measuring payoffs and the ordering of the payoffs accurately in real-world situations where game theory is applicable [17,18], the SD has been taken to be a possible alternative to the PDG in studying emerging cooperative phenomena [16].

The present work will focus on two aspects of current interest. In many circumstances, the connections in a competing population are better modelled by some networks providing limited interactions than a fully connected network. Previous studies showed that different spatial structures might lead to different behaviors [7,8,19–22]. For example, it has been demonstrated that spatial structures would promote cooperation in the PDG [7,8], but would suppress cooperation in the SG [16]. There are other variations on the SG that resulted in improved cooperation [23,24]. Here, we explore the effects of an underlying network on the evolutionary SG in a population in which there exists an additional type of players. The latter is related to the fact that real-world systems usually consist of people who would adopt a strategy other than just C and D. For example, there may be people who do not like to participate in the competition and would rather take a smaller but fixed payoff that is risk-free and fair. In the context of PDG, such persons are called loners. Hauert et al. studied the effects of the presence of such loners [11,12] in a generalization of the PDG called the public goods game (PGG). Motivated by the works of Hauert et al. [11,12,16], we study the effects of a third strategy characterized by risk averse players in the evolutionary SG. These players are referred to as L-players, following the notations in PDG. In our model, evolution or adaptation is built in by allowing players to replace his strategy or character by that of a better-performing connected neighbor. We focus on both the steady state and the dynamics, and study how an underlying network structure affects the emergence of cooperation. It is found that in a fully connected network, the C-players and D-players *cannot* coexist with the L-players. In a square lattice, however, cooperators are easier to survive. Depending on the payoffs, there are situations in which C-players, D-players and L-players can coexist.

In Section 2, the evolutionary SG incorporating an additional character in a population with connections is presented. In Section 3, we present detailed numerical results in fully connected networks and in square