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Physica A 383 (2007) 651–659
PHYSICA A
www.elsevier.com/locate/physa

Promotion of cooperation induced by the interplay between structure and game dynamics

Feng Fu[a,b], Xiaojie Chen[a,b], Lianghuan Liu[a,b], Long Wang[a,b,*]

[a] *Center for Systems and Control, College of Engineering, Peking University, Beijing 100871, China*
[b] *Department of Industrial Engineering and Management, College of Engineering, Peking University, Beijing 100871, China*

Received 29 January 2007
Available online 8 May 2007

Abstract

We consider the coupled dynamics of the adaption of network structure and the evolution of strategies played by individuals occupying the network vertices. We propose a computational model in which each agent plays a n-round Prisoner's Dilemma game with its immediate neighbors, after that, based upon self-interest, partial individuals may punish their defective neighbors by dismissing the social tie to the one who defects the most times, meanwhile seek for a new partner at random from the neighbors of the punished agent. It is found that the promotion of cooperation is attributed to the entangled evolution of individual strategy and network structure. Moreover, we show that the emerging social networks exhibit high heterogeneity and disassortative mixing pattern. For a given average connectivity of the population and the number of rounds, there is a critical value for the fraction of individuals adapting their social interactions, above which cooperators wipe out defectors. Besides, the effects of the average degree, the number of rounds, and the intensity of selection are investigated by extensive numerical simulations. Our results to some extent reflect the underlying mechanism promoting cooperation.

Keywords: Social networks; Network structure adaption; Heterogeneity; Prisoner's Dilemma; Cooperation

1. Introduction

Cooperative behaviors are ubiquitous in real-world, ranging from biological systems to socioeconomic systems. However, the question of how natural selection can lead to cooperation has fascinated evolutionary biologists for several decades. Fortunately, together with classic game theory, evolutionary game theory provides a systematic and convenient framework for understanding the emergence and maintenance of cooperative behaviors among selfish individuals [1,2]. Especially, the Prisoner's Dilemma game (PDG) as a general metaphor for studying the evolution of cooperation has attracted considerable interests [3].

In the original PDG, two players simultaneously decide whether to cooperate (C) or to defect (D). They both receive R upon mutual cooperation and P upon mutual defection. A defector exploiting a C player gets

*Corresponding author. Laboratory for Intelligent Control, Center for Systems and Control, College of Engineering, Peking University, Beijing 100871, China. Fax: +86 10 62754388.
E-mail addresses: fufeng@pku.edu.cn (F. Fu), longwang@pku.edu.cn (L. Wang).

0378-4371/$ - see front matter © 2007 Published by Elsevier B.V.
doi:10.1016/j.physa.2007.04.099

T, and the exploited cooperator receives S, such that $T > R > P > S$ and $2R > T + S$. As a result, it is best to defect regardless of the co-player's decision. Thus, in well-mixed infinite populations, defection is the evolutionarily stable strategy (ESS), even though all individuals would be better off if they cooperated. Thereby this creates the social dilemma, because when everybody defects, the mean population payoff is lower than that when everybody cooperates. In a recent review Nowak suggested five rules for the evolution of cooperation (see Ref. [4] and references therein). Most noteworthy, departure from the well-mixed population scenario, the rule "network reciprocity" conditions the emergence of cooperation among players occupying the network vertices [5]. That is, the benefit-to-cost ratio must exceed the average number of neighbors per individual. Actually, the successful development of network science provides a convenient framework to describe the population structure on which the evolution of cooperation is studied. The vertices represent players, while the edges denote links between players in terms of game dynamical interactions. Furthermore, interactions in real-world network of contacts are heterogeneous, often associated with scale-free (power-law) dependence on the degree distribution, $P(k) \sim k^{-\gamma}$ with $2 < \gamma < 3$. Accordingly, the evolution of cooperation on model networks with features such as lattices [6–9], small-world [10–12], scale-free [13–15], and community structure [16] has been scrutinized. Interestingly, Santos et al. found that scale-free networks provide a unifying framework for the emergency of cooperation [13].

From the best of our knowledge, so far much previous works of games on networks are based on crystallized (static) networks, i.e., the social networks on which the evolution of cooperation is studied are fixed from the outset and not affected by evolutionary dynamics on top of them. However, interaction networks in real-world are continuously evolving ones, rather than static graphs. Indeed, individuals have adaptations on the number, frequency, and duration of their social ties base upon some certain feedback mechanisms. Instead of investigating the evolutionary games on static networks which constitute just one snapshot of the real evolving ones, recently, some researchers proposed that the network structure may co-evolve with the evolutionary game dynamics [17–23]. Interestingly, as pointed out in Refs. [18,19,22], the entangled evolution of individual strategy and network structure constitutes a key mechanism for the sustainability of cooperation in social networks. Therefore, to understand the emergence of cooperative behavior in realistic situations (networks), one should combine strategy evolution with topological evolution. From this perspective, we propose a computational model in which both the adaptation of underlying network of interactions and the evolution of behavioral strategy are taken into account simultaneously. In our model, each agent plays a n-round PDG with its immediate neighbors, after that, based upon self-interest, partial individuals may punish their defective neighbors by dismissing the social tie to the one who defects the most times, meanwhile seek for a new partner at random from the neighbors of the punished agent. We shall show that such individual's local adaptive interactions lead to the situation where cooperators become evolutionarily competitive due to the preference of assortative mixing between cooperators. The remainder of this paper is organized as follows. In the following section, the model is introduced in detail. Section 3 presents the simulation results and discussions. We finally draw conclusions in Section 4.

2. The model

We consider a symmetric two-player game where N individuals engage in the PDG over a network. The total number of edges M is fixed during the evolutionary process. Each individual i plays with its immediate neighbors defined by the underlying network. The neighbor set of individual i is denoted as Ω_i, which is allowed to evolve according to the game results. Let us denote by s_i the strategy of individual i. Player i can follow two simple strategies: cooperation [C, $s_i = (1, 0)^{\mathrm{T}}$] and defection [D, $s_i = (0, 1)^{\mathrm{T}}$] in each round. Following previous studies [24,25], the payoff matrix M has a rescaled form depending on a single parameter,

$$M = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}, \tag{1}$$

where $1 < b < 2$.

In each round, each agent plays the same strategy with all its neighbors, and accumulates the payoff, observing the aggregate payoff and strategy of its neighbors. The total income of the player at the site x can be