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Multifractality and value-at-risk forecasting of exchange rates☆

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HIGHLIGHTS

- We model the high frequency VaR for EUR/USD returns.
- We use a modified version of the multifractal model of asset returns (MMAR).
- We study the out-of-sample forecasting performance of the MMAR model and two alternative models.
- Our dataset consists of 138,418 5-min round-the-clock observations of EUR/USD spot quotes and trading ticks.

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ABSTRACT

This paper addresses market risk prediction for high frequency foreign exchange rates under nonlinear risk scaling behaviour. We use a modified version of the multifractal model of asset returns (MMAR) where trading time is represented by the series of volume ticks. Our dataset consists of 138,418 5-min round-the-clock observations of EUR/USD spot quotes and trading ticks during the period January 5, 2006 to December 31, 2007. Considering fat-tails, long-range dependence as well as scale inconsistency with the MMAR, we derive out-of-sample value-at-risk (VaR) forecasts and compare our approach to historical simulation as well as a benchmark GARCH(1,1) location-scale VaR model. Our findings underline that the multifractal properties in EUR/USD returns in fact have notable risk management implications. The MMAR approach is a parsimonious model which produces admissible VaR forecasts at the 12-h forecast horizon. For the daily horizon, the MMAR outperforms both alternatives based on conditional as well as unconditional coverage statistics.

1. Introduction

With daily estimated turnover in excess of US$1.3 trillion per day according to BIS [1], the EUR/USD spot foreign exchange (FX) rate is the most important currency pair traded in over-the-counter (OTC) spot markets. Given the volatility of these markets in recent years, the management of currency related asset positions is vital to financial intermediaries and international corporations alike. A widely used approach to financial risk measurement is Value-at-Risk (VaR). This approach enables regulators to determine the appropriate amount of risk capital necessary to ensure a financial intermediary is

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immune to the effects of adverse movements in asset prices, and also provides a yardstick for internal management decisions such as risk budgeting and performance evaluation.

An important caveat to VaR estimation is the well-known fact that financial assets returns – especially at higher frequencies – do not display ideal statistical properties. Instead, multifractal or multiscaling return features that are characterized by a form of time-invariance may yield what is observed as fat-tailed returns with long-range dependence (or so-called long memory). Given these well-documented features, it is important to accurately forecast risk levels, which are consistent with the observed return properties.[1] Failure to correctly account for these properties may result in insufficient capital allocations. Systematic underestimation of appropriate levels of risk capital required may even lead to broader system-wide consequences.[2]

In this paper we propose a parsimonious VaR prediction approach that yields improved FX risk forecasts. Typically, VaR is calculated based on daily return data, although doing so ignores the risk – and potential losses – associated with the liquidation of positions due to adverse intraday price movements. We therefore use intraday data to support our VaR forecasts and employ a modified version of the multifractal model of asset returns (MMAR) proposed by Mandelbrot et al. [14]. The MMAR approach has the benefit that it parsimoniously addresses the complex properties of financial returns. It also allows the incorporation of various degrees of long memory at different powers of returns, while accommodating the presence of fat-tails, which are both stylized facts of financial returns (see e.g. [15–19]). Alternate approaches, such as fractionally integrated GARCH (generalized autoregressive conditional heteroskedasticity) models (or FIGARCH) have the same decay rate for all moments and are not scale-consistent. Moreover, the MMAR was previously found to be a suitable model for FX rate returns.[3]

Following Clark [21], several studies have argued that trading volume could be utilized to improve risk prediction. For example, King et al. [22] investigate the relationship between USD/CAD returns and order flow and argue that trading volume has strong out-of-sample predictive power for USD/CAD returns. Xue and Gençay [19] demonstrate that the existence of different market traders, with multiple trading frequencies, can increase volatility persistence. Intraday asset volatility also varies with the number of market traders. Given this evidence, we model MMAR trading time by the series of trading volume ticks and provide a modified MMAR approach for out-of-sample VaR forecasting. We thereby overcome limitations of previous multifractal model applications such as their combinatorial nature and their restriction to a bounded interval.[4]

In order to test the forecasting ability of our novel VaR approach, we study the out-of-sample accuracy of VaR predictions for both 12-h and daily (24-h) forecast horizons. While these forecast periods are somewhat arbitrary they are consistent with the trading activities expected of global financial intermediaries with a subsidiary, or branch, that is always open during the 24-h trading day. Our high frequency dataset consists of round-the-clock EUR/USD spot exchange rate prices quoted by market participants on the Reuters trading platform during the period January 5, 2006 to December 31, 2007. These prices are bundled into 5-min time stamped intervals with the spot price and the trading ticks recorded. We find that the EUR/USD returns are multifractal, with the moments showing different scaling exponents. Our MMAR approach is then compared with forecasts based on historical simulation and a benchmark location-scale VaR model based on GARCH(1,1). The results show that the MMAR approach produces admissible VaR forecasts for the 12-h forecast horizon. For the daily horizon, we find that the MMAR outperforms both alternatives based on conditional as well as unconditional coverage statistics.

Besides this investigation, there are several other studies that predict out-of-sample intraday VaR. Giot [26], for example, uses GARCH models with normal and Student-*t* innovations and RiskMetrics model for modelling intraday VaR of 15- and 30-min returns of three stocks traded on the NYSE. The results show that a superior model is based on Student-*t* innovations. Sun et al. [27] try to take account of the stylized facts of 1-min frequency DAX returns by using a GARCH model with Lévy stable and normal innovations. The authors find that the model with Lévy stable innovations outperforms the competing intraday VaR models. Dionne et al. [28] analyse a high frequency sample consisting of 63 trading days of three stocks traded on Toronto Stock Exchange. Their backtesting results imply that a logarithmic autoregressive, conditional duration, exponential GARCH model achieves better intraday VaR forecasts than ordinary GARCH and historical simulation. In contrast to these studies, our approach is not based on GARCH volatility and our MMAR VaR model is able to capture the stylized facts of intraday data, including leptokurtosis and long-range dependence. An alternative group of studies in the area deals with extreme value theory (EVT) and VaR forecasting. Gençay and Selçuk [29], Gençay et al. [30], Wagner [31] as well as Maghyereh and Al-Zoubi [32] for example, investigate the performance of VaR models for daily stock returns based on EVT. The papers

[1] The empirical evidence is extensive. For example, Calvet and Fisher [2] find multifractality in a (Deutsche Mark) DMK/USD high frequency series and Xu and Gençay [3] also prove 5-min USD/DMK returns are multifractal. Nekhili et al. [4] investigate scale properties of US Dollar/Deutsche Mark FX returns and state that the co-existence of short-term as well as long-term traders indicates different time scales for different market traders. Eisler and Kertész [5] report multiscaling behaviour for a high frequency stock index series and high frequency observations of the 200 most liquid stocks at New York Stock Exchange (NYSE). Fillol [6] suggests a model for replicating the scaling properties observed in the French CAC-40 (Cotation Assistée en Continu) stock series. Mulligan and Koppl [7] find long-range dependence in US macroeconomic data.

[2] This is especially important for emerging markets, where domestic political instability (e.g. [8]) may lead to region-wide contagion effects, and eventually broader-world wide problems. To some extent these risks can be reduced through diversification [9,10]. Note that recent evidence using fractal based measures links the efficiency of financial markets to their level of institutional development [11–13].

[3] Calvet and Fisher [20] find that MMAR outperforms both GARCH and FIGARCH as models of foreign exchange rate series.

[4] Calvet and Fisher [23] overcome these shortcomings by introducing a Markov-switching multifractal model, while Lux [24] provides a further model alternative. Note that McCulloch [25] models the intraday trading time using a unifractal rather than multifractal time.